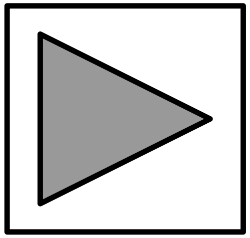
Limit Cycle Walking and Running of Biped Robots

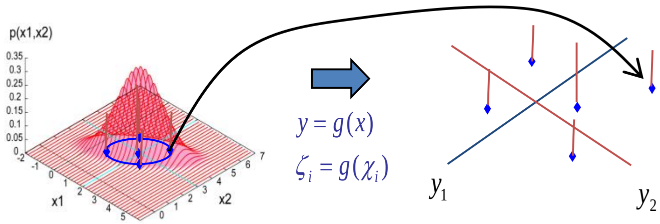
Masaki Yamakita
Tokyo Institute of Technology

Introduction of Yamakita Lab.



Other Research Topics

- **State estimation of stochastic nonlinear systems**



- **Application of nonlinear model predictive control(NMPC)（Robot, Engine）**
- **Parallel computation algorithm of NMPC**
- **Identification of nonlinear/hybrid systems**
- **Optimal adaptive control**

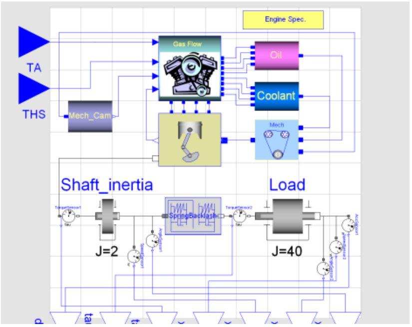
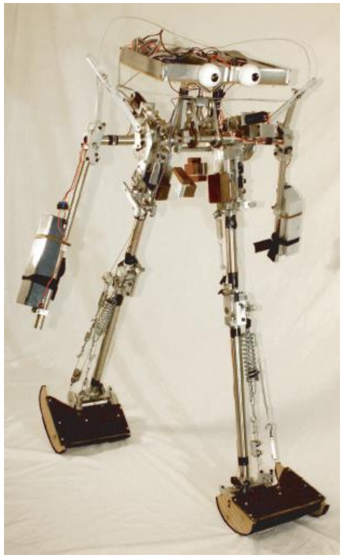


Table of presentation

Biped walking robots

ASIMO(Honda)

HRP4 (AIST)



Cornel robot (Collins et al.)

Biped robots have been studied as a mobile robot that can walk on various environments.

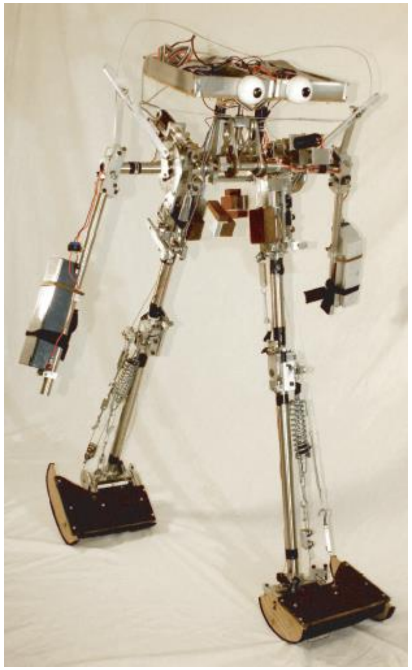
They can walk on paved road, uneven surface, stairs and so on.

In the resent years, **limit cycle walkers** have also been studied as an energy-efficient biped robot.

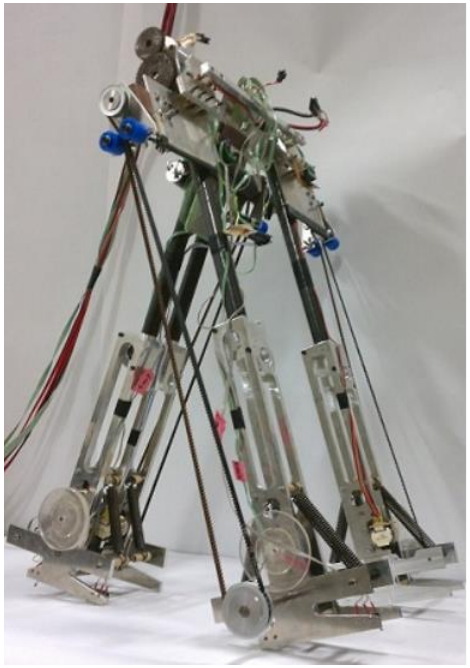
Limit cycle walking robots

Many limit cycle walkers have been developed.
They achieve energy-efficient and stable walking
by simple control methods.

Delft robot ``Denise”
(Wisse et al.)



Cornel robot
(Collins et al.)



Our robot ``DASU-walker”
(Hanazawa et al.)

ASIMO vs Limit cycle walkers

Limit cycle walking is generally slow.

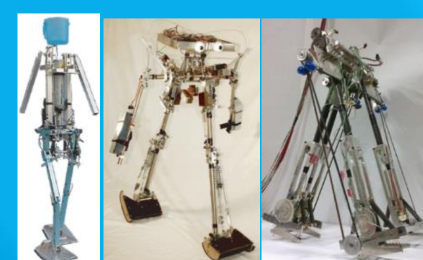
Good

Walk Speed

ASIMO
(ZMP based walker)

Limit cycle walkers are more efficient than ASIMO.
However, many limit cycle walkers are slower than ASIMO.

Limit cycle walkers



BAD

Energy-efficiency

Good

Desired limit cycle walker

Good

Walk Speed

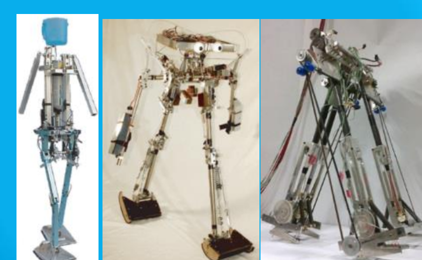
ASIMO
(ZMP based walker)

Desired
limit cycle walker

?

We want to achieve fast and efficient limit cycle walking!!

Limit cycle walkers



BAD

Energy-efficiency

Good

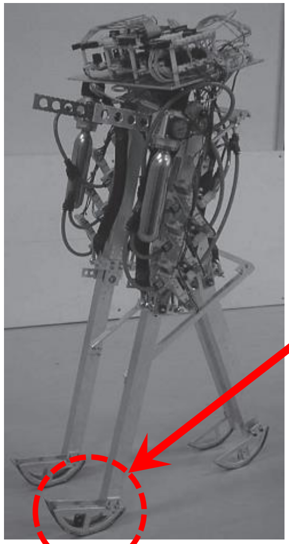
Our Proposed Solutions

- Introduction of inerters in addition to conventional impedance at ankles

- Introduction of wobbling mass in a body

Limit Cycle Walking of Arc-Shaped Biped Robots

Blue-biped
(Nagoya Institute of Technology)



Arc-footed biped walker
(Osaka Univ.)

To achieve high-speed and high-efficient walking, many latest biped robots have arc-shaped feet.

- ➢Arc-shaped foot
- ➢Without ankle

Asano et al. show analytically that arc-footed biped robots have a mechanism to improve walking speed and energy efficiency.(2006)

Flat-Footed Robots with Mechanical Impedance

Flat-footed passive walker
(Keio Univ.)

Flame
(Delft Univ.)

- ➢Flat-foot
- ➢Ankle spring
- ➢Ankle damping

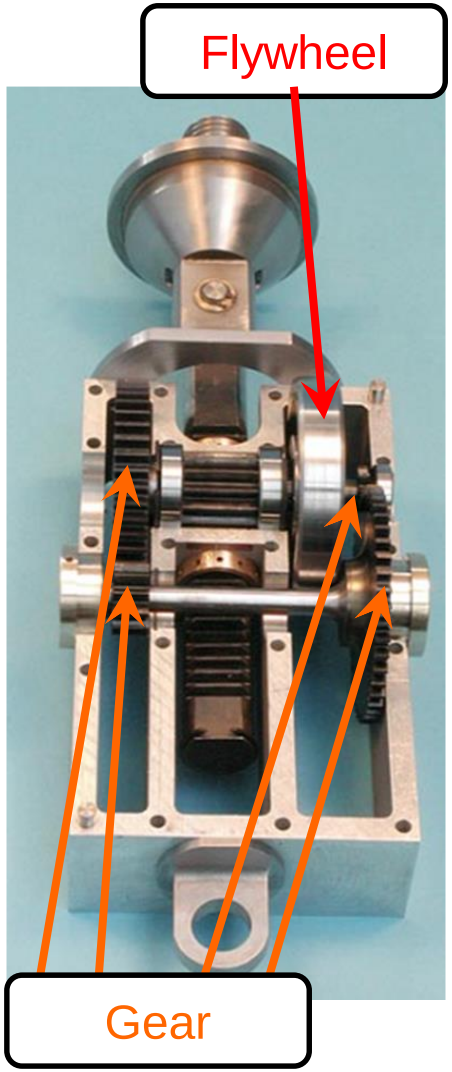
Arc-footed robots have some problems

- ➢ The robots have difficulty to keep static state.
- ➢ The robots cannot realize push-off, running and jumping by ankle actuation.

To solve these problems and achieve excellent biped walking, flat-footed biped robots with mechanical impedance at ankle were proposed.

In the walking of these robots, <u>ankle elasticity</u> is known important element to determine walking performance.

Mechanism to Easily Design Inertia



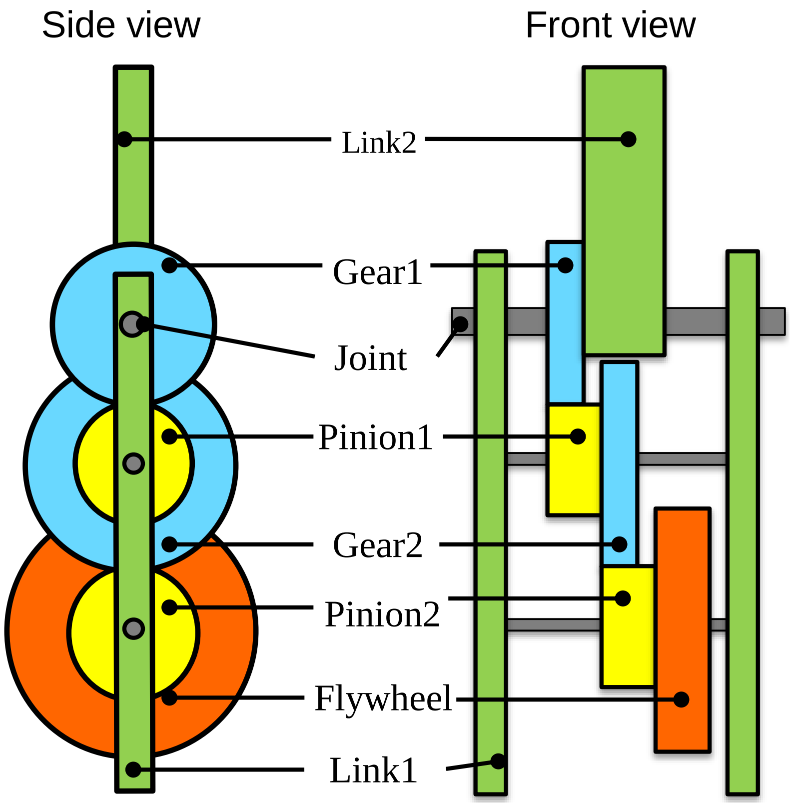
Inerter
(Cambridge Univ.)

From point of view of mechanical impedance, we consider that the biped robots achieve more high-speed and high-efficient walking by optimization of ankle elasticity, viscosity and inertia for biped walking.

We cannot easily design ankle inertia since the ankle inertia of the robots have design restriction such as foot size and weight.

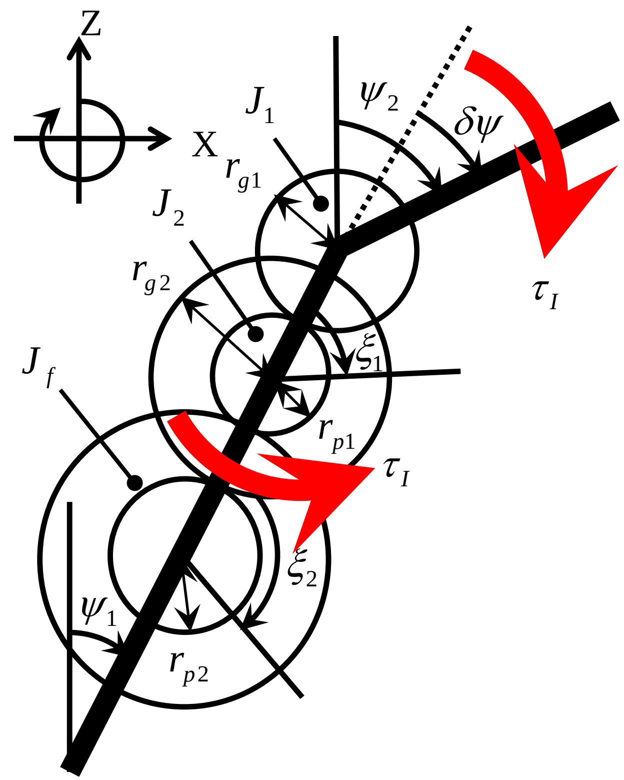
As mechanism of designing inertia, Inerter is proposed by Smith.
We can easily design inertia of the Inerter by changing gear ratio and flywheel,

Mechanism of Rotary Inerter for Biped Robots



Mechanism of the rotary inerter

Modeling



Model of the rotary inerter

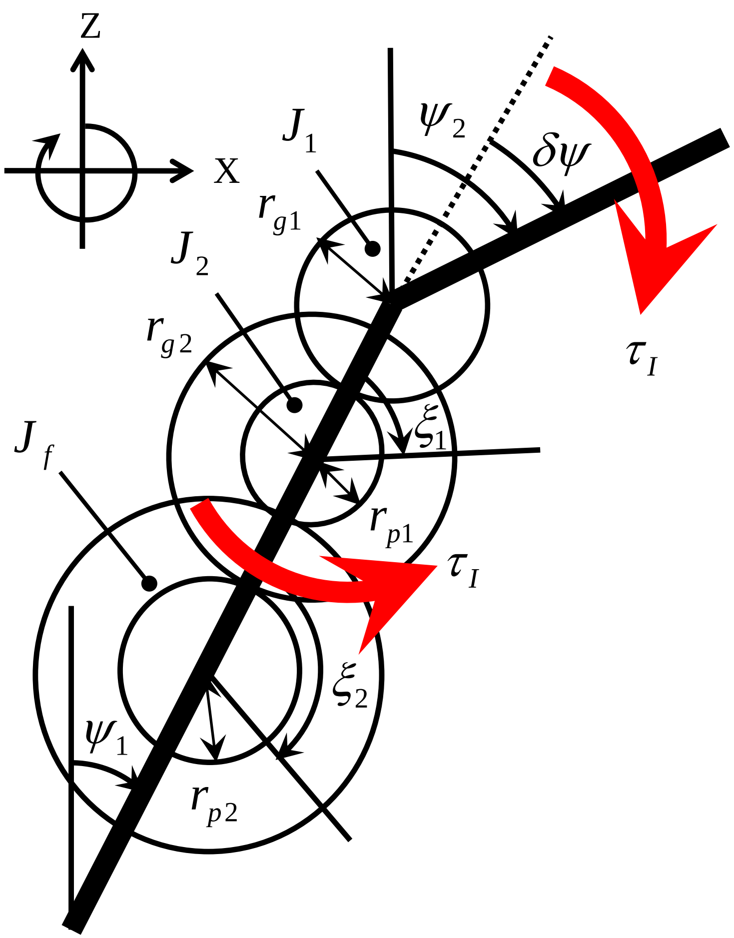
Rotary inerter has
two links, two gears,
two pinions and a flywheel.

Dynamic equation

$$\tau_I = \beta(\ddot{\Psi}_2 - \ddot{\Psi}_1)$$

where β is constant due to
the gear ratio and the flywheel inertia

Derivation of Dynamic Equation of the Rotary Inerter



Model of the rotary inerter

We derive three dynamic equations

$$J_1\ddot{\psi}_2 = \tau_I - r_{g1}F_1$$

$$J_2\ddot{\xi}_1 = r_{p1}F_1 - r_{g2}F_2$$

$$(J_3 + J_f)\ddot{\xi}_2 = r_{p2}F_2$$

J_i : *Inertia of gear etc.*
r_{pi} : *Radius of pinion.*
r_{gi} : *Radius of gear.*
ψ_i : *Link angle.*
ξ_i : *Rotation angle of gear.*
F_i : *Force between gear and pinion*

We arrange above three equations

$$\tau_I = J_f\left(\frac{r_{g1}r_{g2}}{r_{p1}r_{p2}}\right)^2(\ddot{\psi}_2 - \ddot{\psi}_1)$$

We define that rg_1/rp_1 is α_1 and rg_2/rp_2 is α_2

$$\tau_I = \boxed{J_f\alpha_1^2\alpha_2^2}(\ddot{\psi}_2 - \ddot{\psi}_1)$$

Inerter Constant : $\beta = \boxed{J_f}\boxed{\alpha_1^2\ \alpha_2^2}$

Inertia of the flywheel　Gear ratio of Inerter

Movie of Ankle Inerter

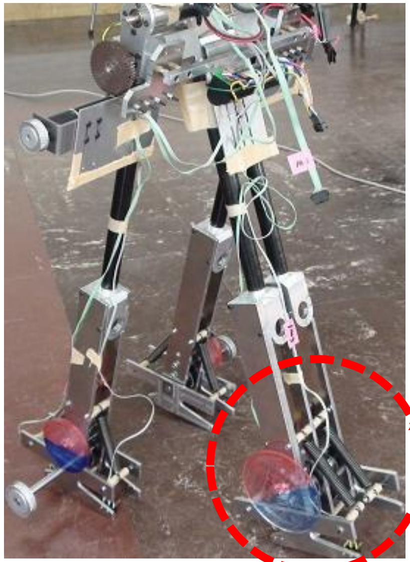
Without Inerter

With Inerter

We can see that effect of the Inerter.

Ankle Inerter

To easily design mechanical impedance at ankle
for high-speed and high-efficient biped walking,
we proposed a new flat-footed biped robot
with an ankle spring, damping, and inerter.



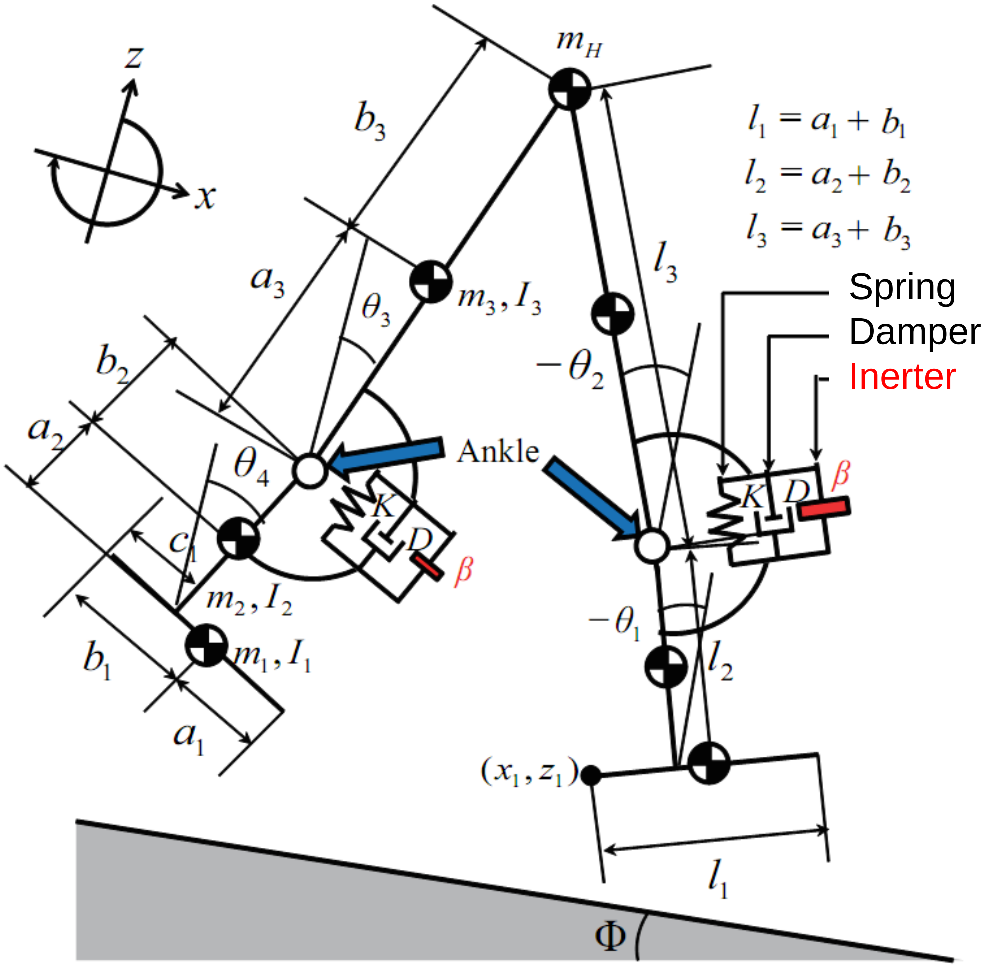
Our biped walker

- Flat-foot
- Ankle spring
- Ankle damping
- Ankle inerter

Ankle Inerter

To show effectiveness of the proposed method,
we present simulation and experiment results
of passive dynamic walking of the robot.

Walking Model



Walking model

q is a position vector

$$\mathbf{q} = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad x_1 \quad z_1]^T$$

Link angle — Heel position

The robot model has flat feet and three elements of mechanical impedance at ankles

Dynamic Equation

$$\mathbf{M}_\mathrm{I}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = -\mathbf{K}_\mathrm{m}\mathbf{q} - \mathbf{D}_\mathrm{m}\dot{\mathbf{q}} + \mathbf{J}_\mathrm{C}^\mathrm{T}\boldsymbol{\lambda}$$

where inertia term of $\mathbf{M}_\mathbf{I}$ includes inertia of the ankle inerter

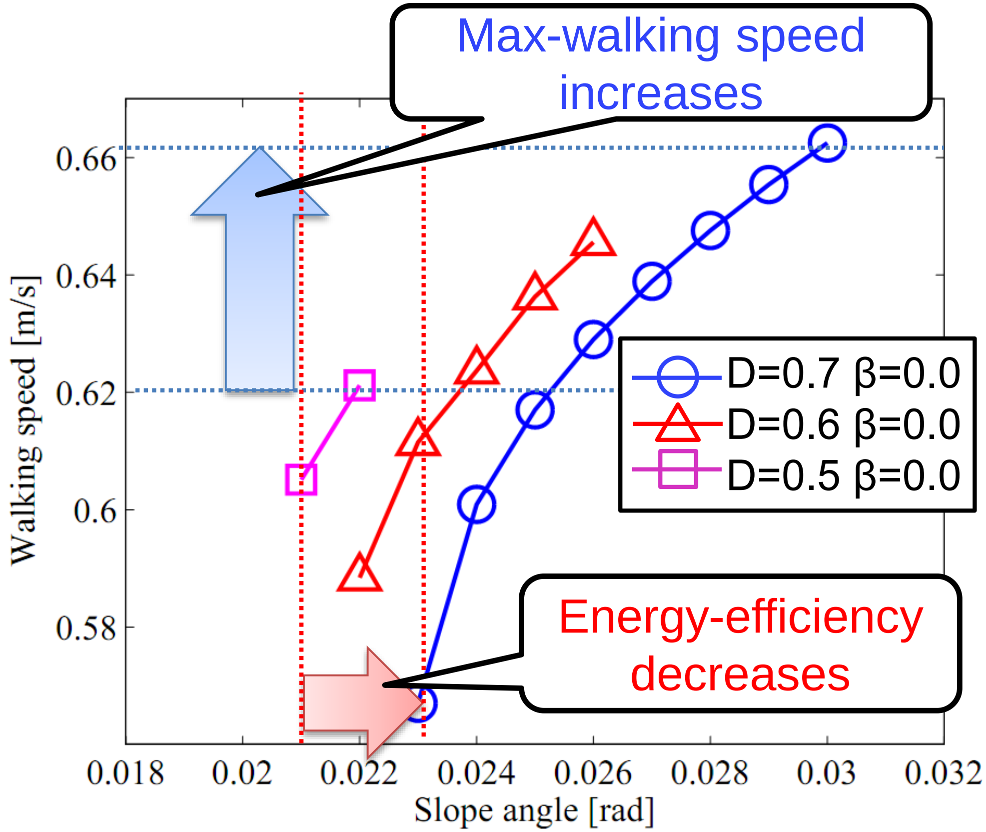
$$\mathbf{M}_\mathrm{I} := \mathbf{M}_0 + \boldsymbol{\beta}$$

Inertia term without inerter

Inertia by inerter

Effect of Ankle Viscosity in Biped Walking

Walking speed with respect to slope angle



※ Slope angle is equal to Specific Resistance (SR)

$$SR := \frac{p}{Mgv} = \frac{MgX_g \sin\phi / T}{Mgv} = \sin\phi \approx \boxed{\phi}$$

PDW is energy-efficient when slope angle is small.

To show effect of ankle viscosity in biped walking, we set different ankle viscosity D [Nm/(rad/s)] of the biped robot. we carry out simulation in each case.

When ankle viscosity D increases,

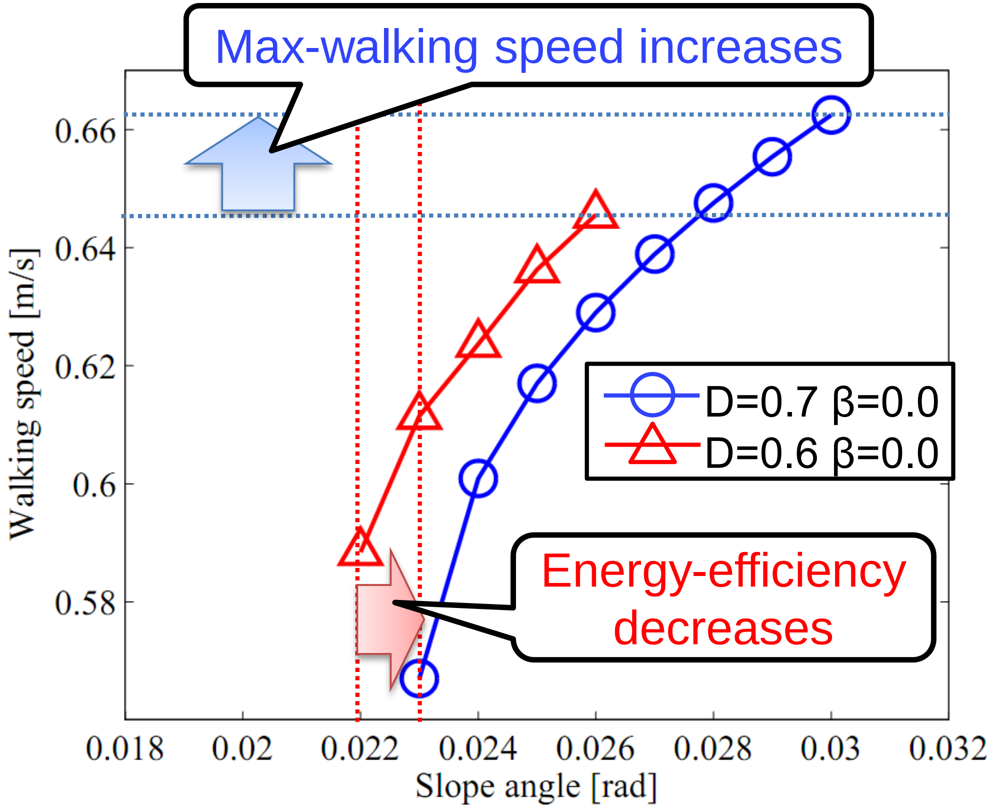
- Max-walking speed increases.
- Energy-efficiency decreases.

Ankle viscosity determine

- Walking speed
- Energy efficiency

Trade-off of Walking Performance

Walking speed with respect to slope angle



To improve max walking speed of the biped robot, we change only ankle <u>viscosity</u> for high-speed walking.

D=0.6 Nm/(rad/s) β=0.0 Nm/(rad/s^2)

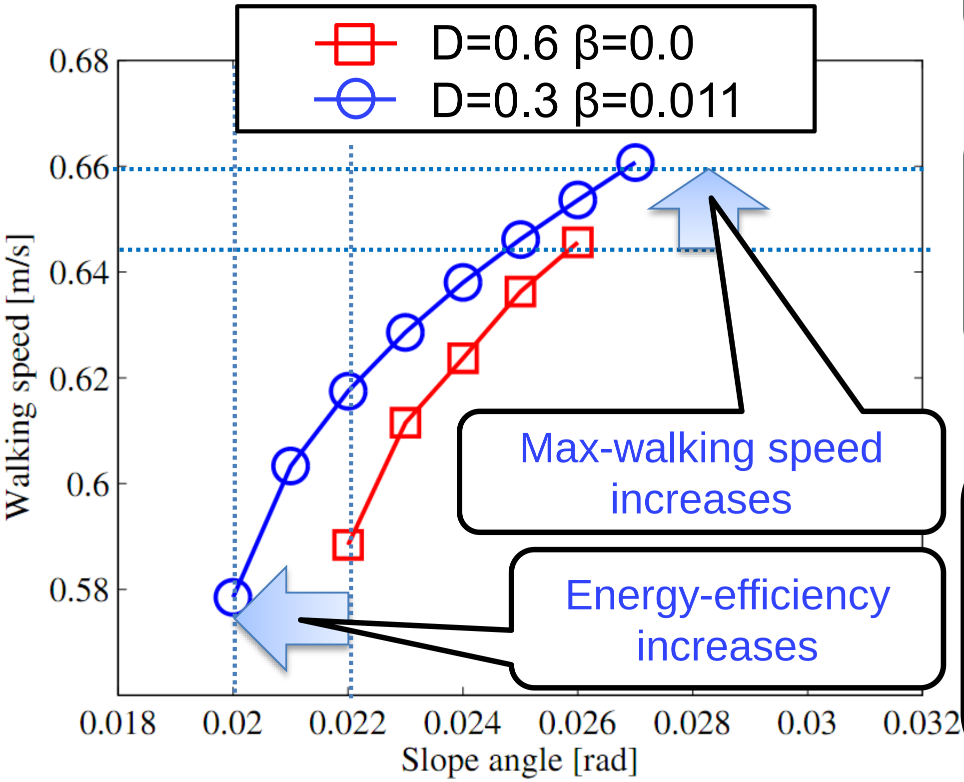
D=0.7 Nm/(rad/s) β=0.0 Nm/(rad/s^2)

- Max-walking speed increases.
- Energy-efficiency decreases.

We cannot improve both walking speed and energy-efficiency by design of ankle viscosity.

Achieving Excellent Walking by Ankle Inerter

Walking speed with respect to slope angle



In the biped robot with ankle inerter, we can easily design ankle inertia by the ankle inerter (β [Nm/(rad/s)]).

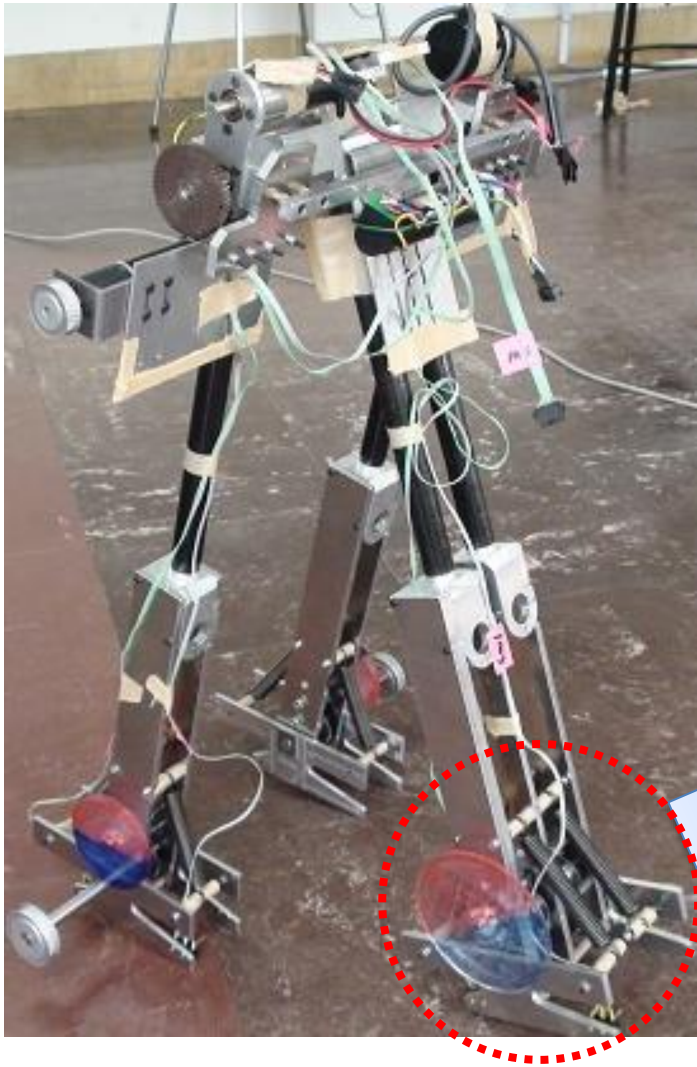
We consider improving walking speed by design of ankle viscosity and inertia for high-speed walking without decreasing energy efficiency,

We change ankle viscosity and inertia.
D=0.6 Nm/(rad/s) β=0.0 Nm/(rad/s^2)

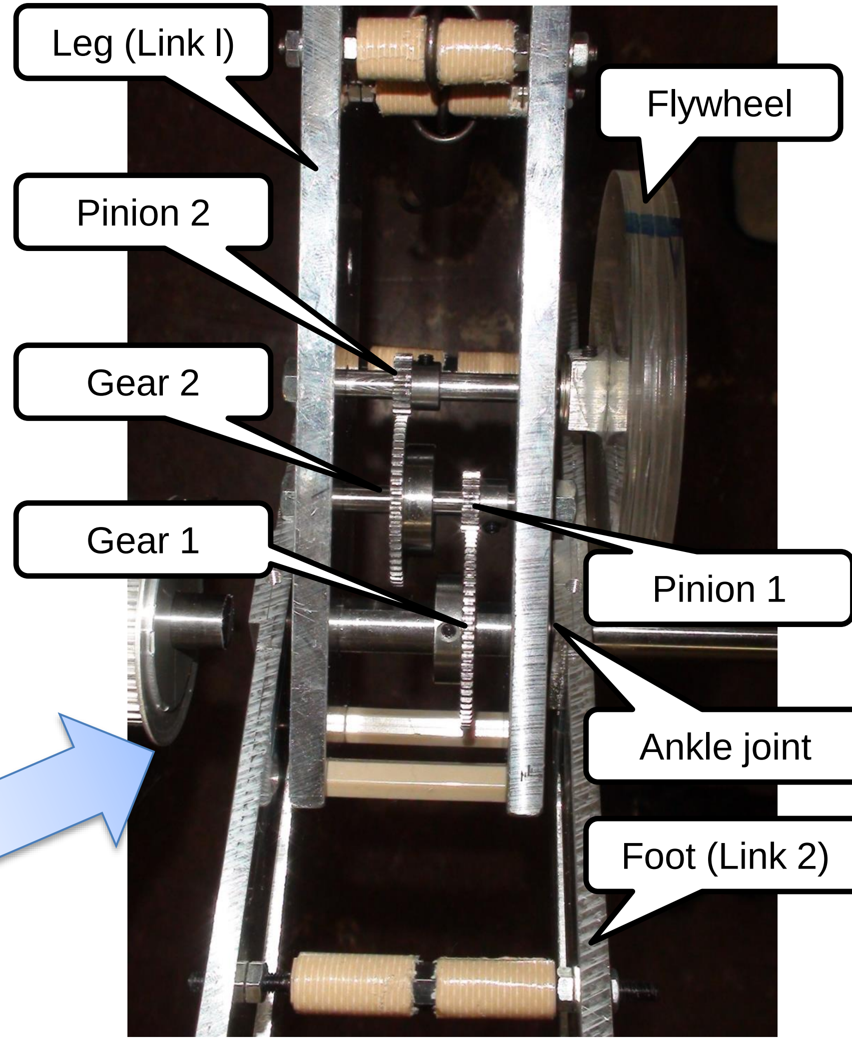
D=0.3 Nm/(rad/s) β=0.011 Nm/(rad/s^2)

We can improve both walking speed and energy-efficiency by design of ankle viscosity and Inertia.

Passive Dynamic Walker with Ankle Inerter



Flat-footed passive dynamic walker with ankle inerter



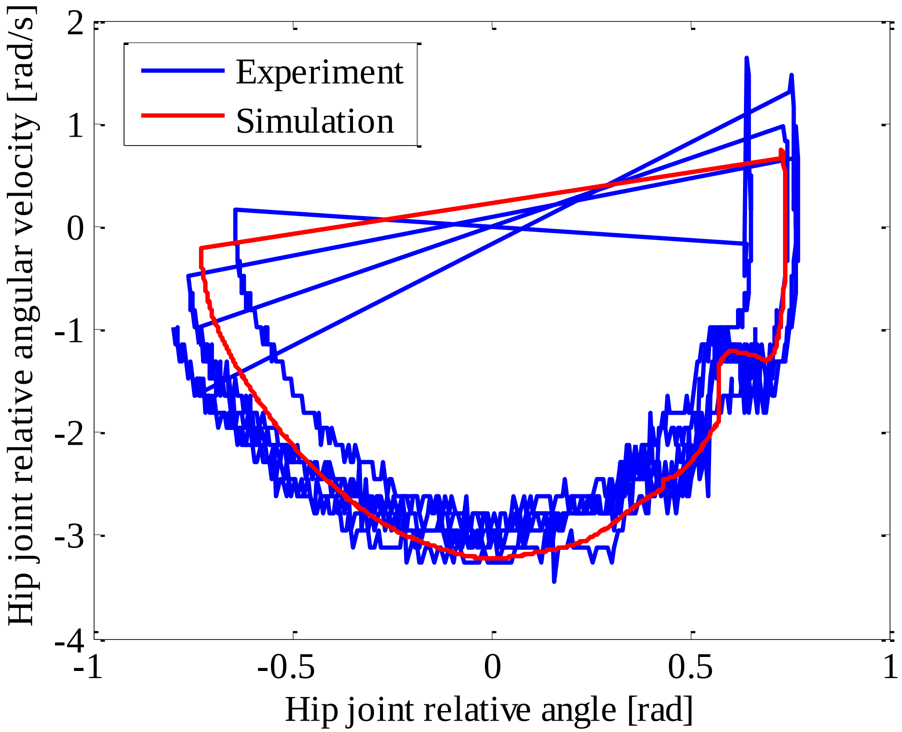
Rotary Inerter at ankle

Walking Experiment

Our biped robot achieves stable 1-period passive walking

Experiment and Simulation Result

Phase plane trajectory of the hip joint



Walking speed and period

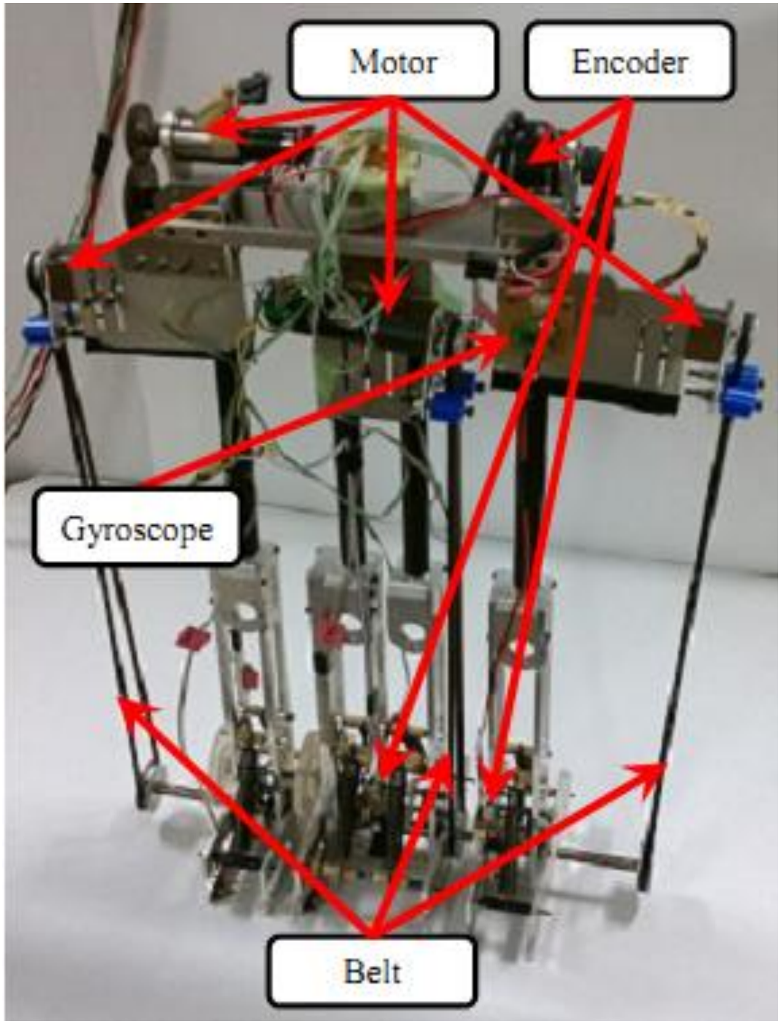
	Simulation	Experiment
Speed[m/s]	0.54	0.53
Period[s]	0.76	0.77

Biped walker achieves walking which is equal to the simulation result

Our biped walker model and the simulation results are valid.

Passive to Active Control

Overview

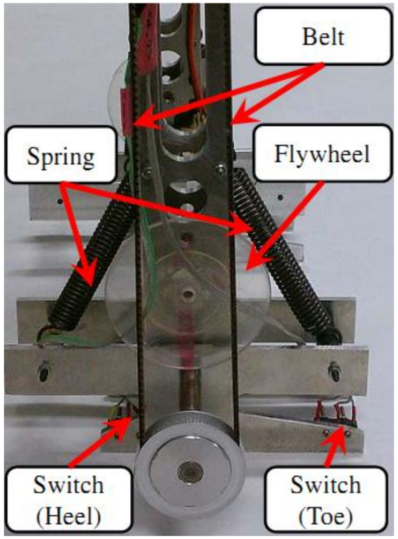


Height = 0.6 m
Weight = 6.7 kg

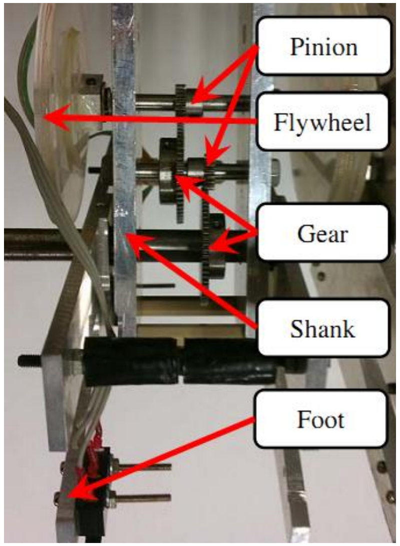
Modification of the robot

- **4 actuators are added**
- **Foot shape is flat, and inerters and springs are installed at ankles**
- **gyro meter and encoders are added**

Side view of the foot

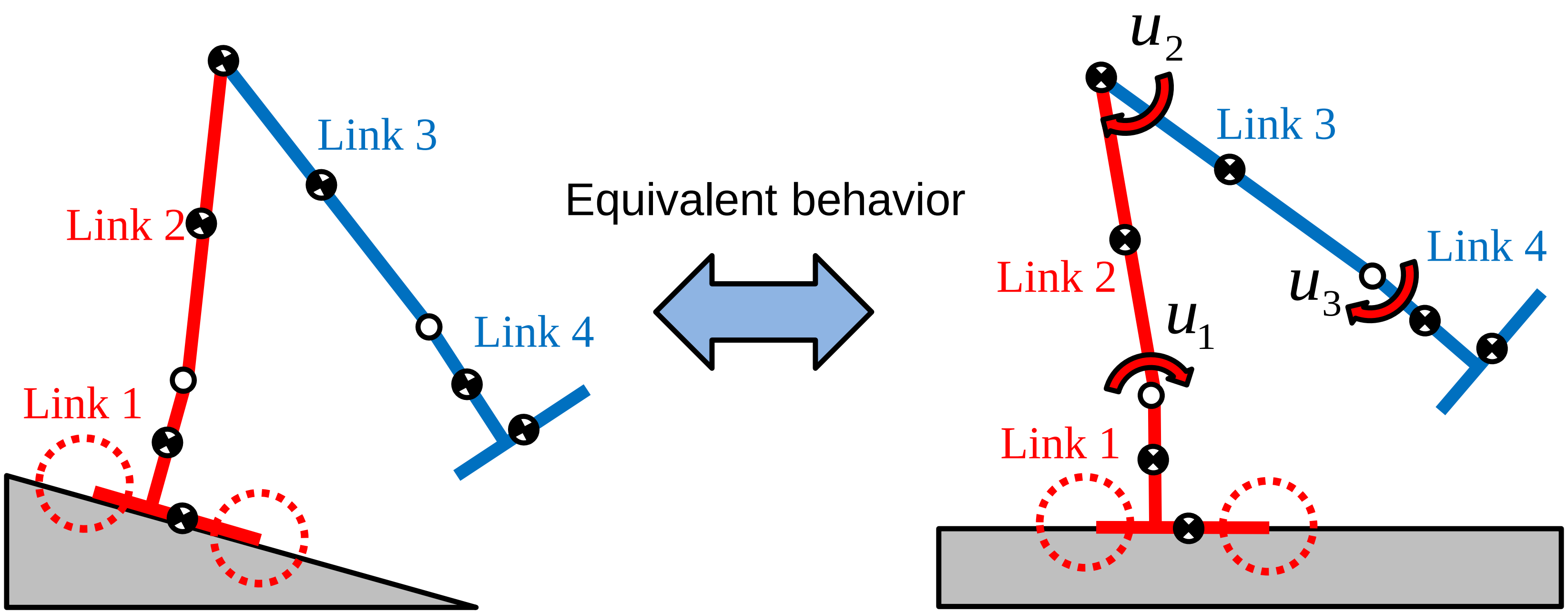


Front view of the foot



Control Algorithm

To mimic the passive dynamic walking,
Potential energy shaping(PES) method is extended.
→Virtual gravity is generated by the actuators



$$u = B_{3L}^{-1}(G_{3L}(\theta_{3L}) - G_{3L}^{\mathrm{d}}(\theta_{3L}))$$

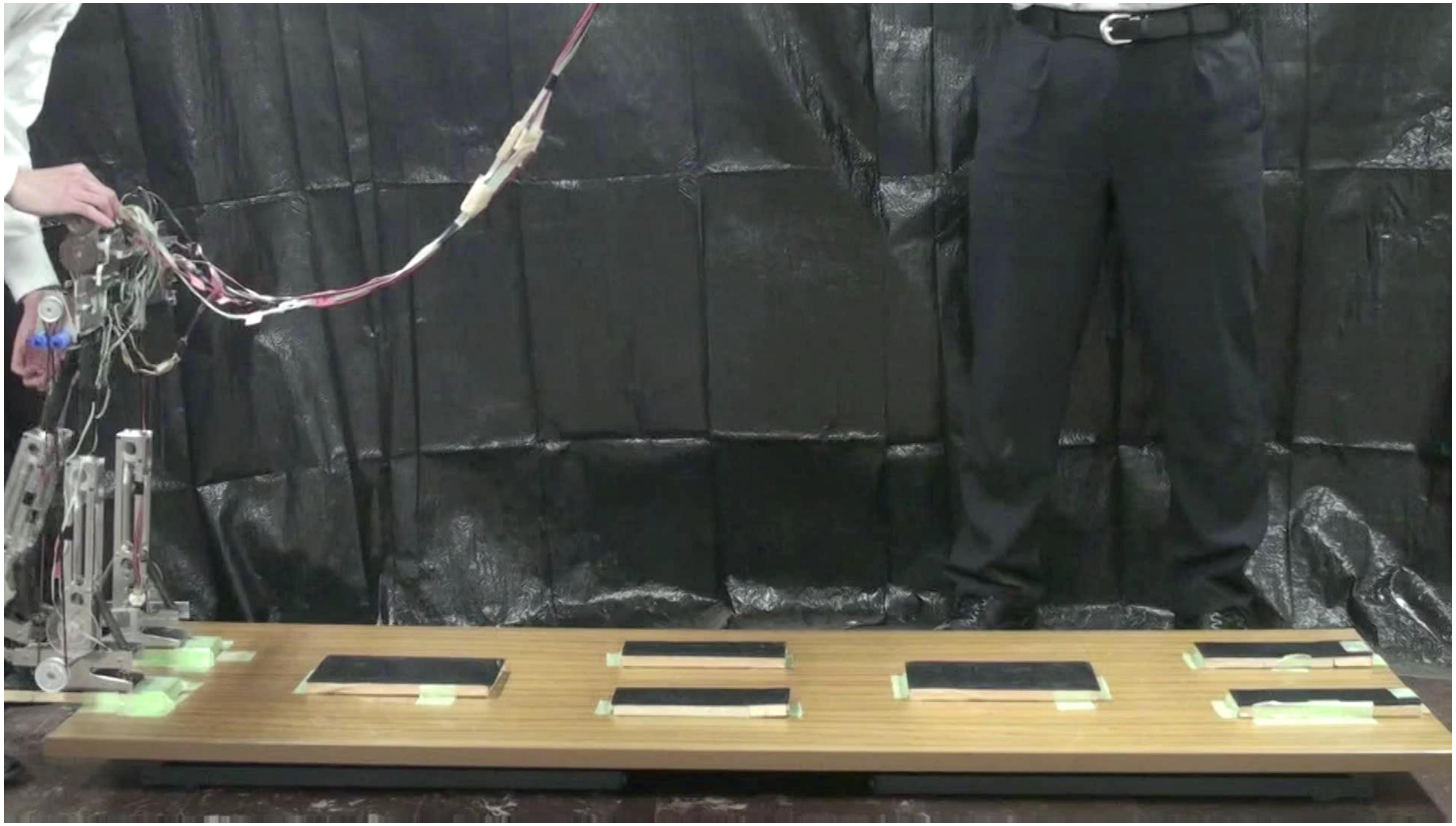
Conversion matrix

Gravity terms on level ground of 2nd, 3rd, 4th elemtns

Gravity terms on the slope of 2nd , 3rd, 4th elements

Experimental Result

Periodic walking was realized
(with some modification of control gains)



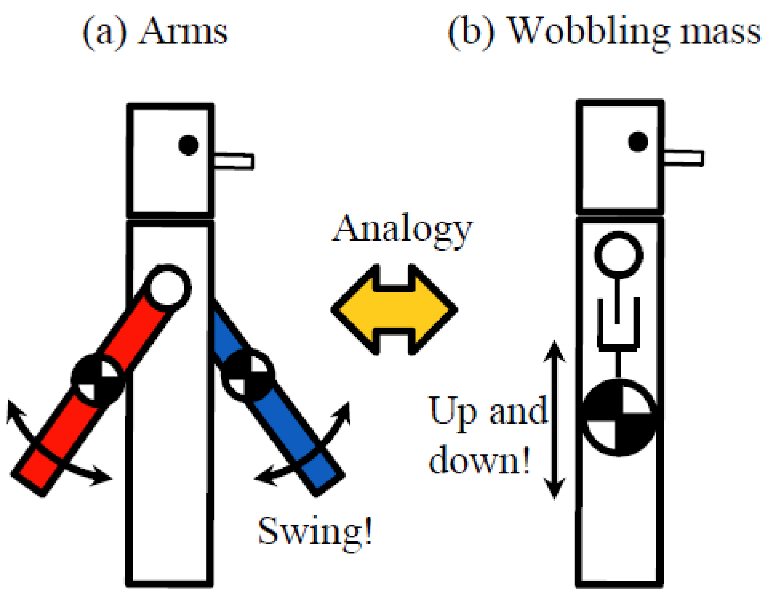
Obtained Result 1

We showed following contents

- Biped robot with the ankle inerter achieves excellent walking by design of ankle viscosity and inertia.
- Mechanism of our biped robot with an ankle inerter.
- By walking experiment, our simulation results are valid.

By ankle inerter, flat-footed biped robots with mechanical impedance at ankles can achieve more high-speed and high-efficient walking.

Fast and efficient human walking



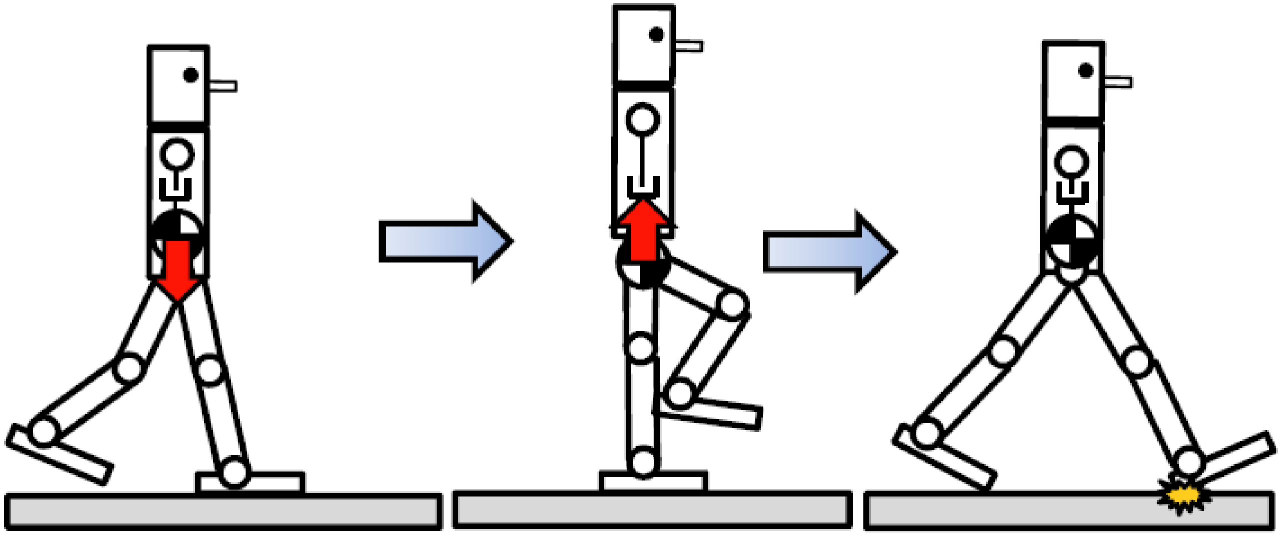
To get the novel speeding-up method,
we consider fast and efficient **human walking**.

We notice that an **analogy** between **swinging arms** and **up-and-down a mass**.

From this analogy, we infer that humans achieve fast dynamic walking by **active up-and down motion of the arm mass.**

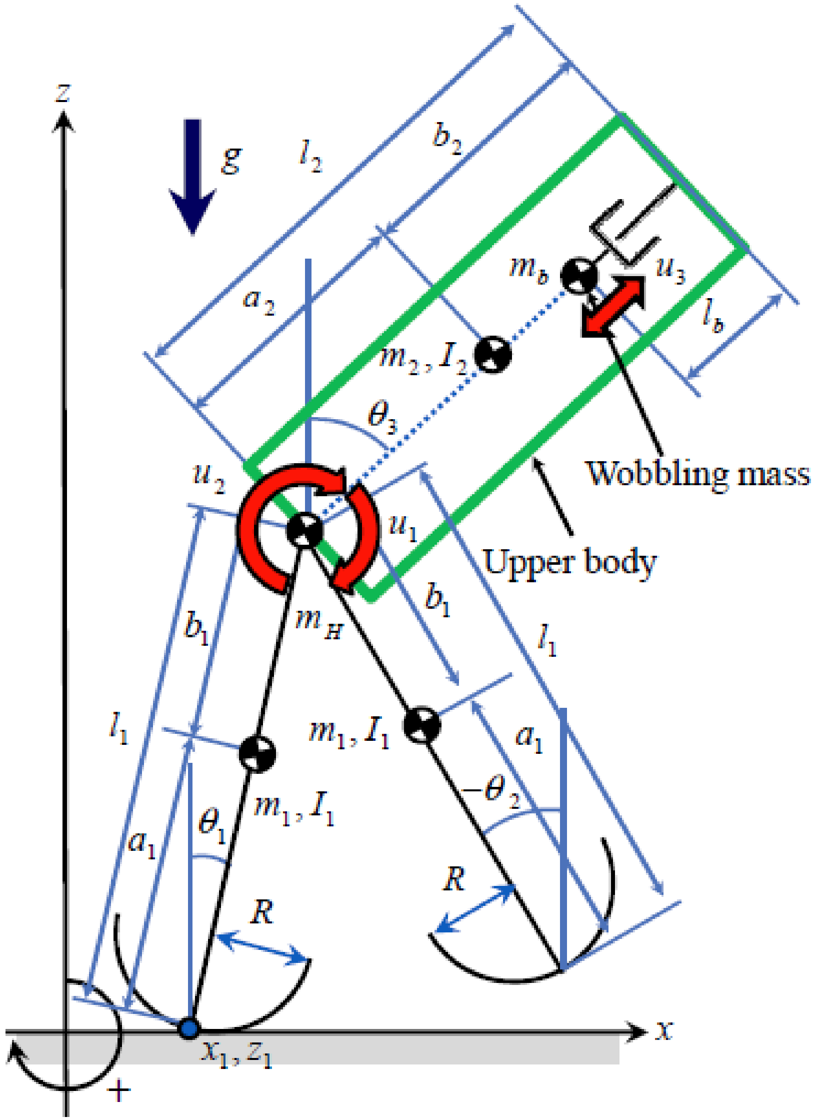
Active up-and-down motion of the wobbling mass

We present a novel speeding-up method
for limit cycle walkers
using **active up-and-down motion of a wobbling mass**



We show the effectiveness of the proposed method
by numerical and mathematical results.

Model of robot with wobbling mass



Generalized coordinate vector

$$q = [\theta_1 \quad \theta_2 \quad \theta_3 \quad l_b \quad x_1 \quad z_1]^{\mathrm{T}}$$

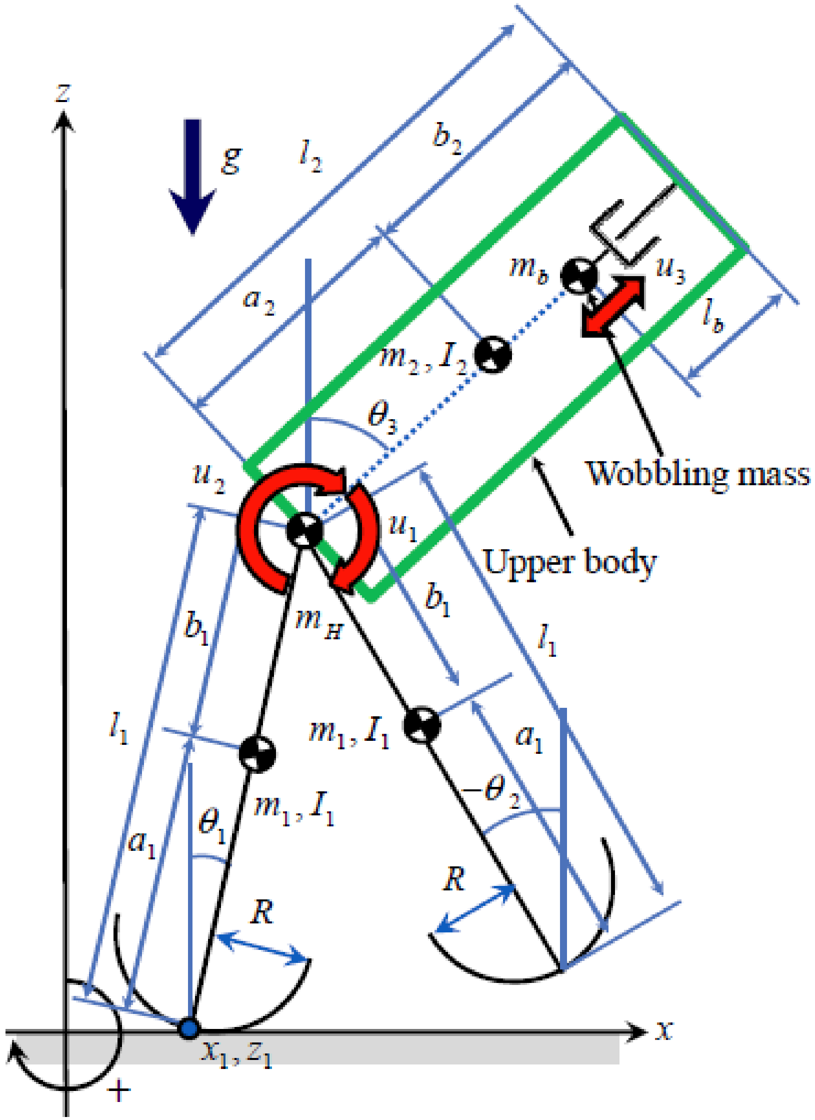
Dynamic equation

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) = \mathbf{S}_1\mathbf{u} + \mathbf{J}_{\mathrm{C}}^{\mathrm{T}}\boldsymbol{\lambda}$$

Mechanical parameters

Leg and torso length = 1.0 [m]
Leg and torso weight = 5.0 [kg]
Hip weight = 5.0 [kg]
Wobbling mass weight = 2.5 [kg]
Arc-feet radius = 0.1 [m]

Control method for fast limit cycle walking①



Input for swing leg control

$$u_1 = -K_{P1}(\phi_d - (\theta_1 - \theta_2)) - K_{D1}(\dot{\theta}_2 - \dot{\theta}_1)$$

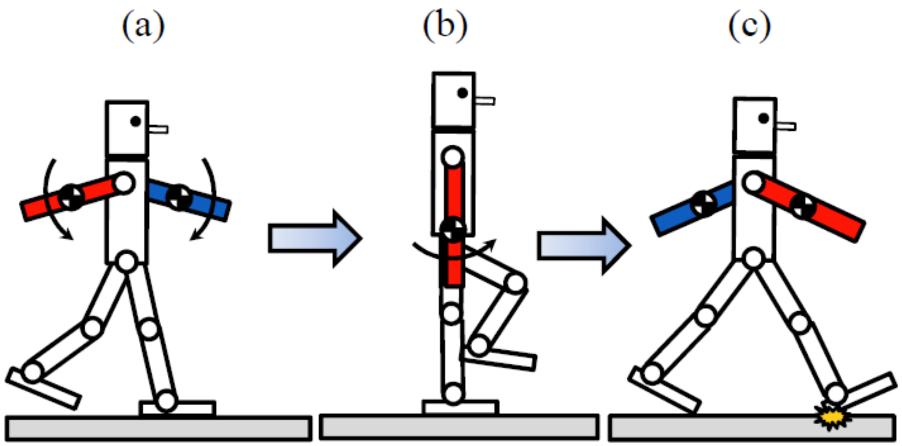
Robot swings up the swing leg

Input for torso control

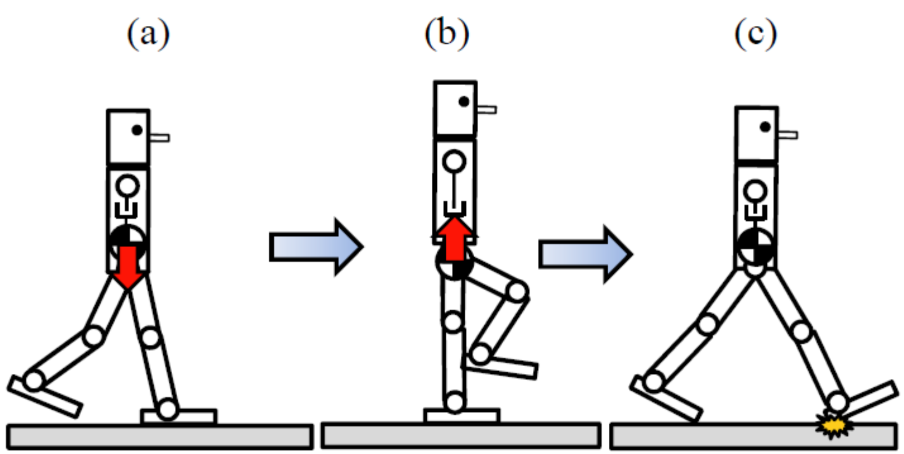
$$u_2 = -K_{P2}(\theta_3 - \theta_{3d}) - K_{D2}\dot{\theta}_3 + u_1$$

Robot maintains the torso angle

Control method for fast limit cycle walking②



Dynamic walking with swing arms



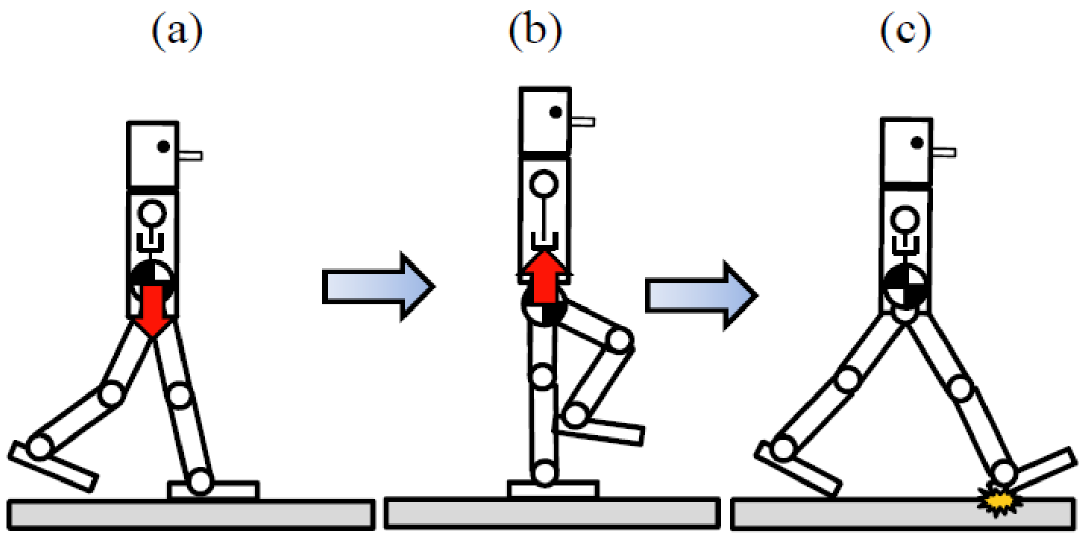
Dynamic walking with wobbling mass

We consider a control method of a wobbling mass

The up-and-down of arm mass is antiphase with respect to up-and-down of the torso mass.

We design the control method achieving **antiphase up-and down mass motion with respect to up-and-down the torso.**

Control method for fast limit cycle walking③



Height of the torso mass

Input for wobbling mass

$$u_3 = -K_{P3}(l_b - l_{bd}) - K_{D3}(\dot{l}_b - \dot{l}_{bd})$$

where $l_{bd} = k(p_{tz} - p_0)$ is the desire trajectory for wobbling mass

This desire trajectory is the **antiphase trajectory with respect to up-and-down motion of the torso mass**.

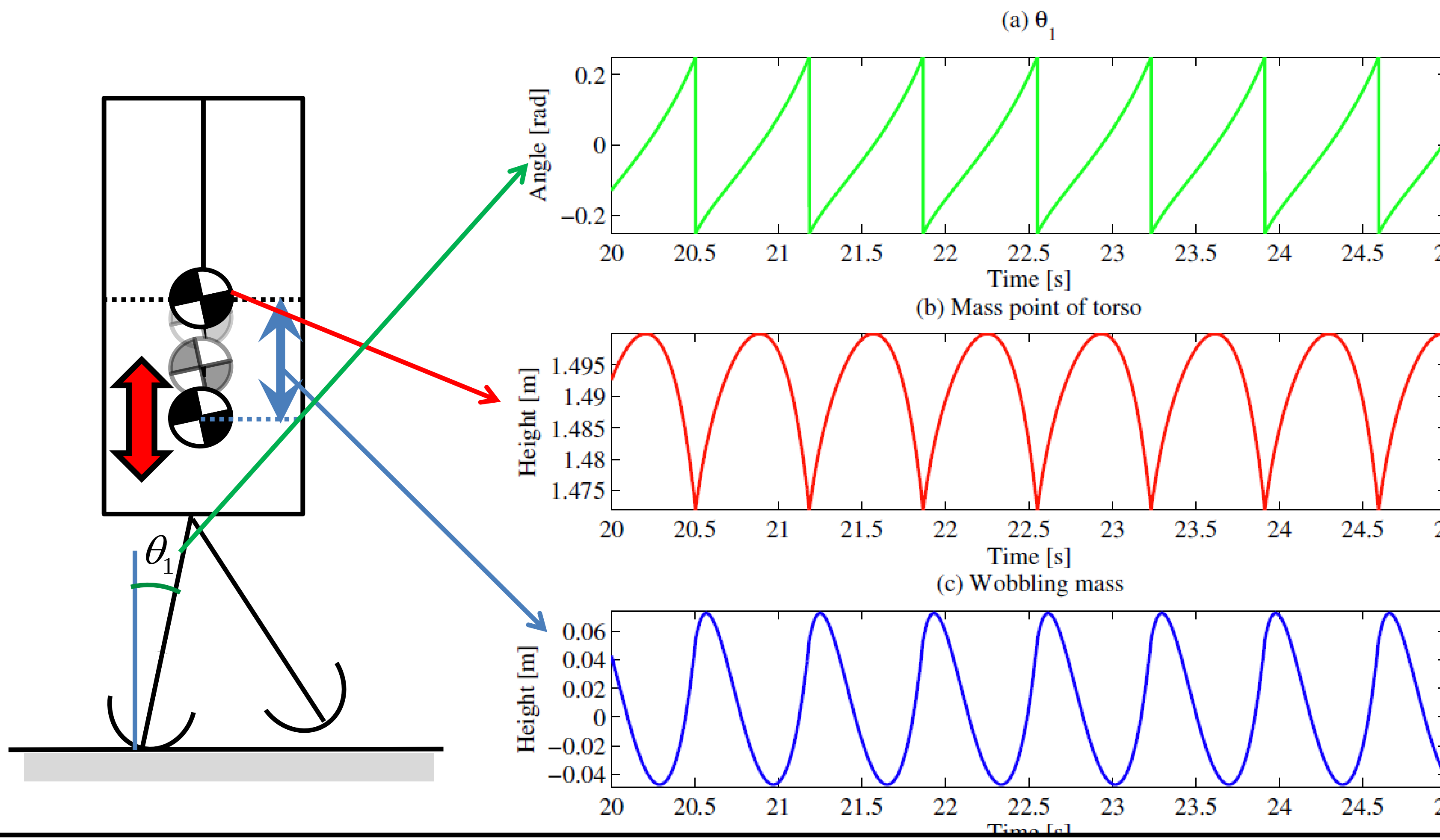
Numerical results①

This video shows 3 types biped walking

1. Walking without mass, 2. Walking with locked mass,

3. Walking with active controlled mass

Numerical result②

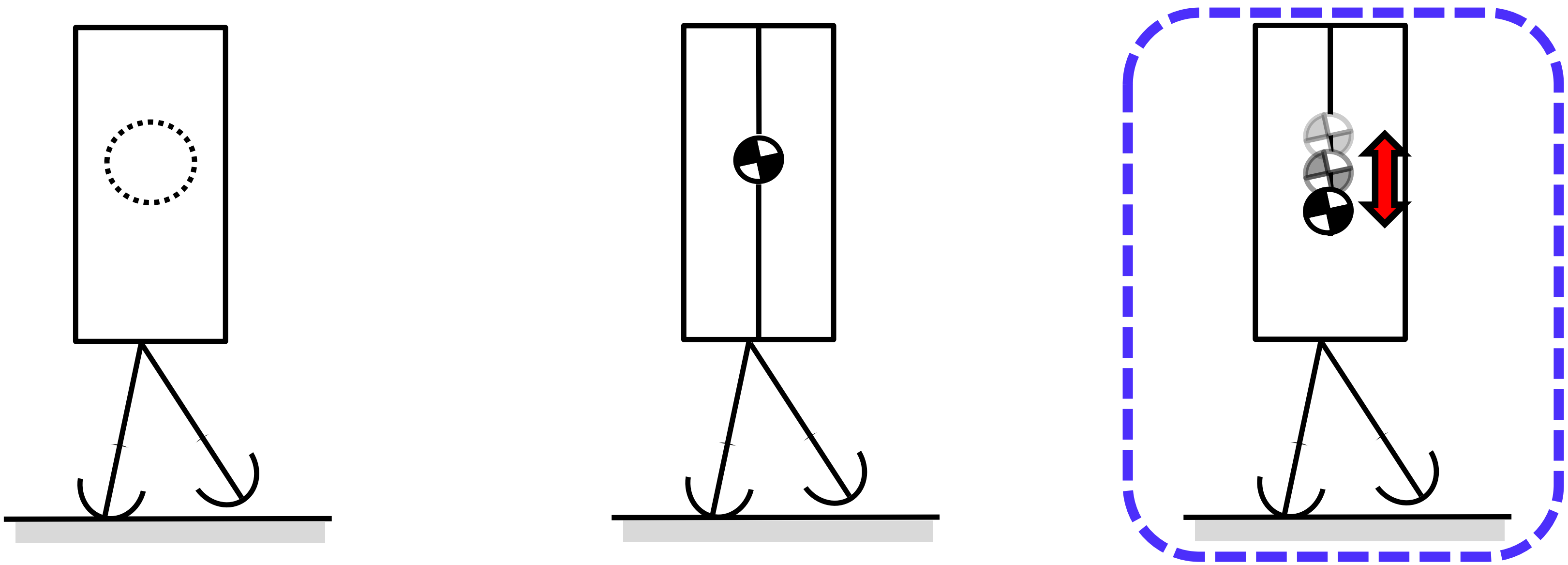


Proposed control achieves
desired up-and-down motion of the wobbling mass

Numerical results③

Speed in each walking type

Walking type	**Walking speed** [m/s]
without wobbling mass	0.59
with locked wobbling mass	0.51
with active wobbling mass (proposed)	$\boxed{0.73}$ (SR = 0.2 [-])



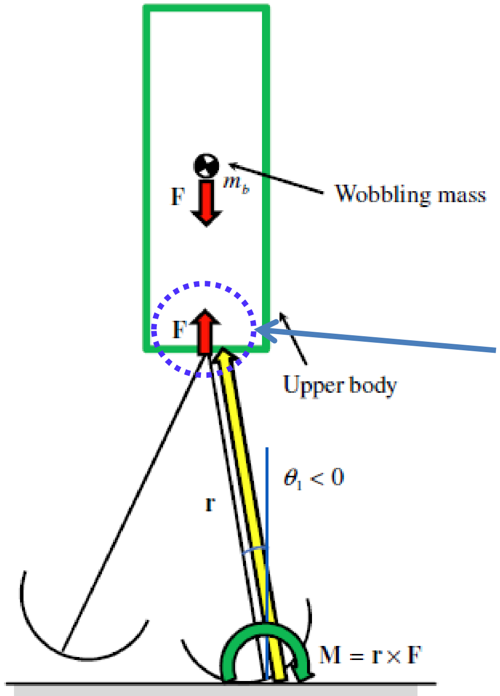
Biped robot achieves fast walking by the proposed method

Mathematical analysis①

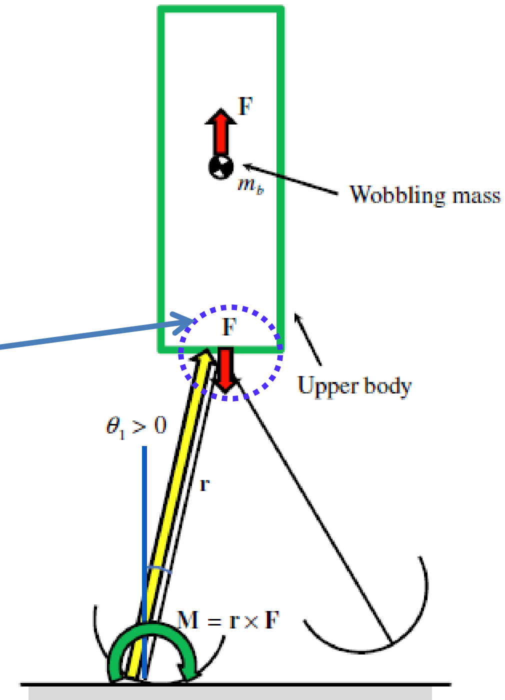
To more clearly show the effectiveness of the proposed method, we have **mathematically analyzed the limit cycle walking**.

When the stance leg angle is negative, the wobbling mass goes down by the proposed control.

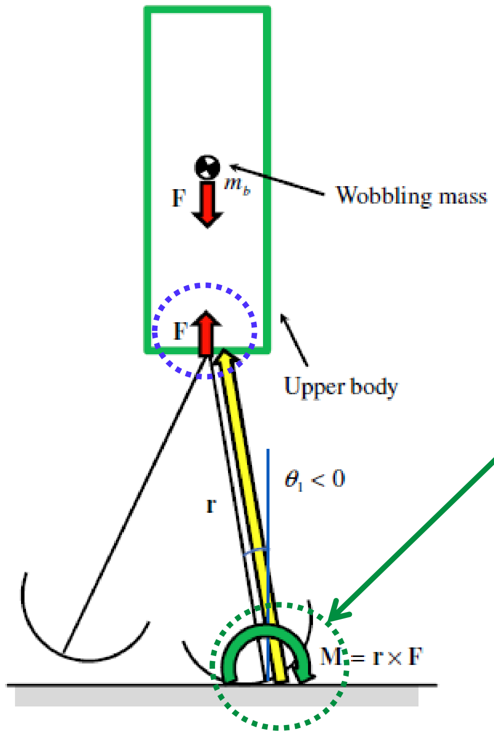
When the stance leg angle is positive, the wobbling mass goes up by the proposed control.



Up-and-down motion of the wobbling mass generates Reaction force.

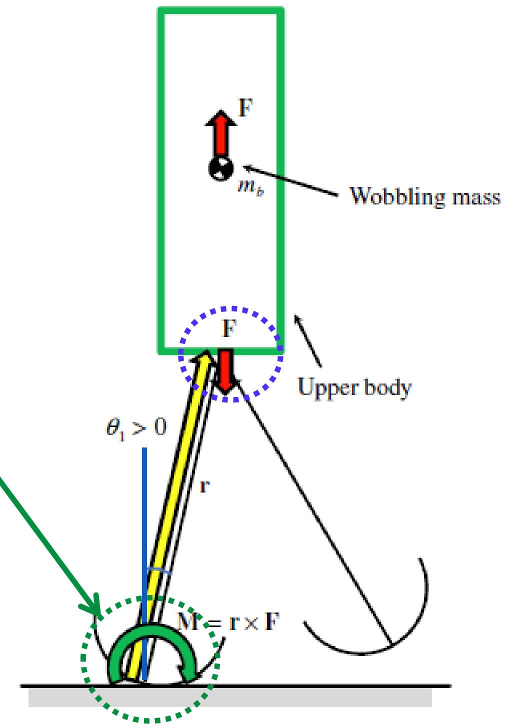


Mathematical analysis②



Moment due to the reaction force is given by

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



Energy due to this moment is given by

$$E = \int_{T_s}^{T_0} \underbrace{\dot{\theta}_1(-F l_1 \theta_1)}_{\text{positive}} dt + \int_{T_0}^{T_e} \underbrace{\dot{\theta}_1(F l_1 \theta_1)}_{\text{positive}} dt$$

Biped robot gets positive energy.

Proposed method generates **driving torque** like ankle torque.

Obtained Result 2

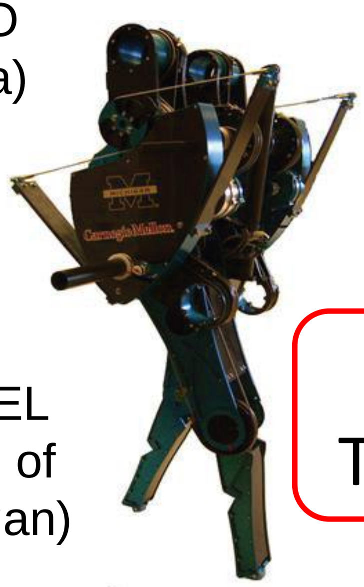
We have proposed the novel speeding-up method for limit cycle walking using an actively controlled wobbling mass.

We have shown that the biped robot achieves fast limit cycle walking by active up-and-down motion of the wobbling mass.

We have mathematically shown that **our proposed method generates driving torque like ankle torque**.

Running Robots

ASIMO
(Honda)



MABEL
(Univ. of
Michigan)

Conventional running robots

➡ To realize stable running

Relatively slow speed

Max. speed

Human : 6～10 [m/s]

Robots :2～3[m/s]

Challenge !

To realize stable and fast running robots

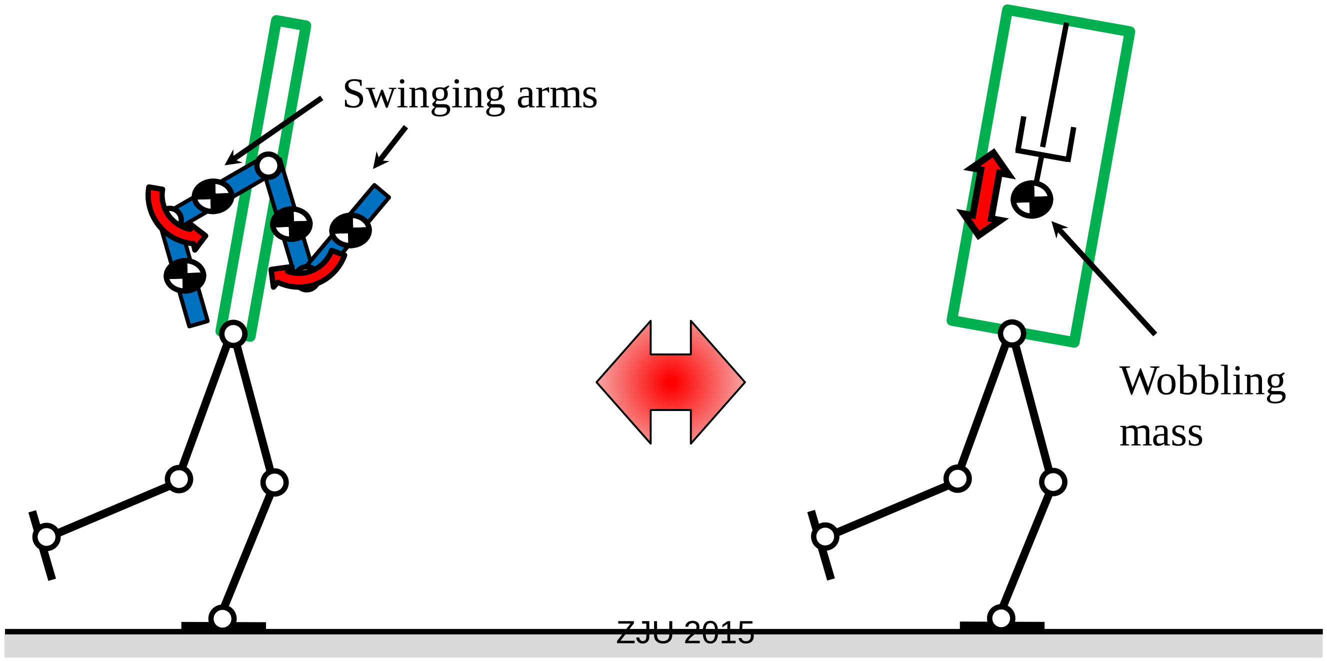
Human also use Swinging Arms when Running

Human uses swinging arms when running

Effects of swinging arms is mimicked by wobbling mass

Faster running is expected

Verification by numerical simulations



Model of Robot

Generalized coordinate vector

$$\boldsymbol{q} = [\theta_1,\ \theta_2,\ \theta_3,\ \theta_4,\ \theta_5,\ \theta_6,\ \theta_7,\ l_b,\ x_1,\ z_1]^{\mathrm{T}}$$

Dynamic equation

$$\boldsymbol{M}(\boldsymbol{q})\ddot{q} + \boldsymbol{C}(\boldsymbol{q},\dot{q})\dot{q} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{S}_1\boldsymbol{u} + \boldsymbol{S}_2\boldsymbol{\tau} + \boldsymbol{J}_c(\boldsymbol{q})^{\mathrm{T}}\boldsymbol{\lambda}$$

From stance phase to flight

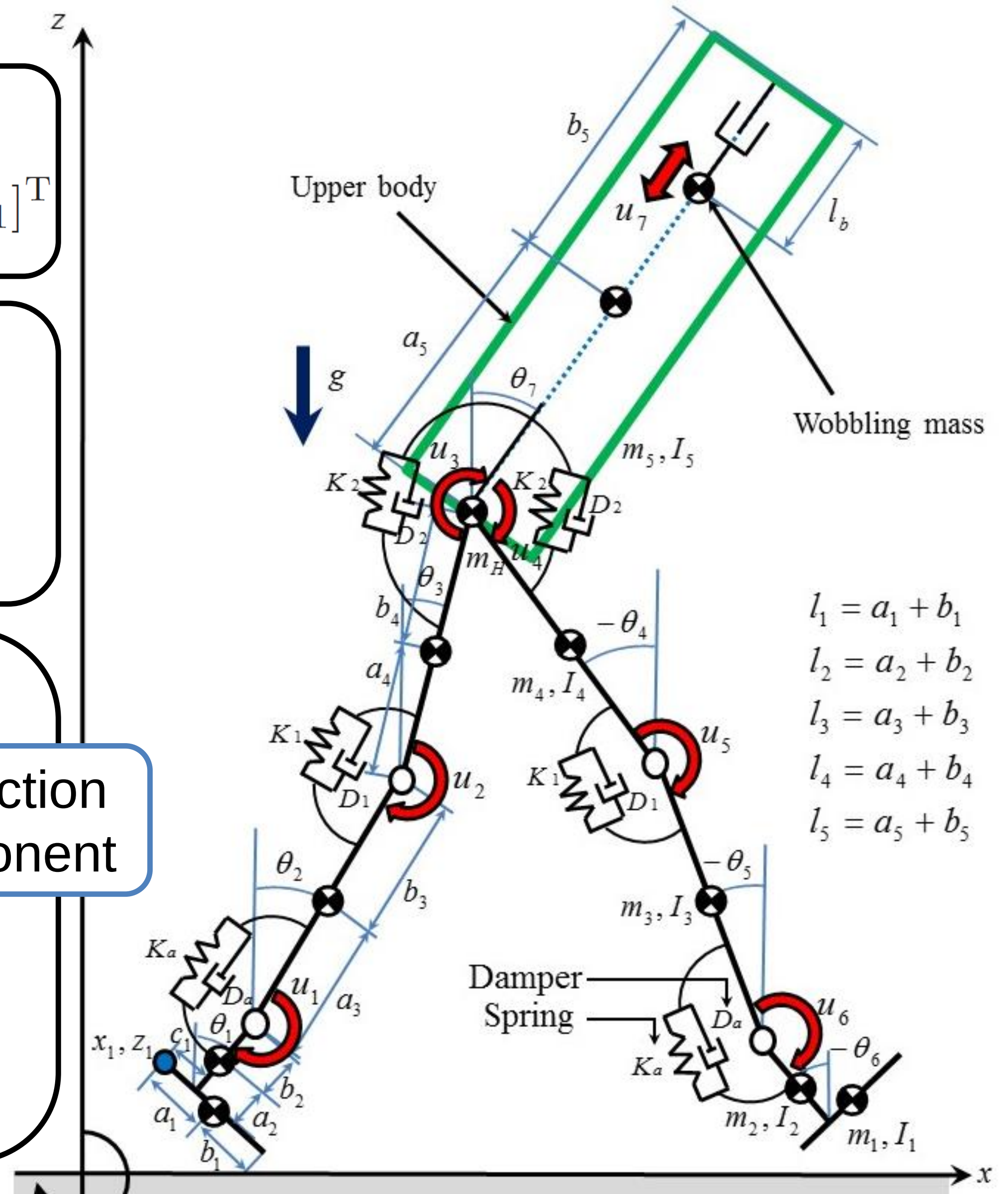
$$F^z_{t_{st}} = 0$$

Vertical reaction force component

From flight phase to stance

$$p^z_{h_{sw}} = 0$$

Height of swinging heel



1.2 SLIP Model(Spring loaded invert pendulum)

Running based on SLIP model is lasting ballistic jumping using spring energy

- Stance pha

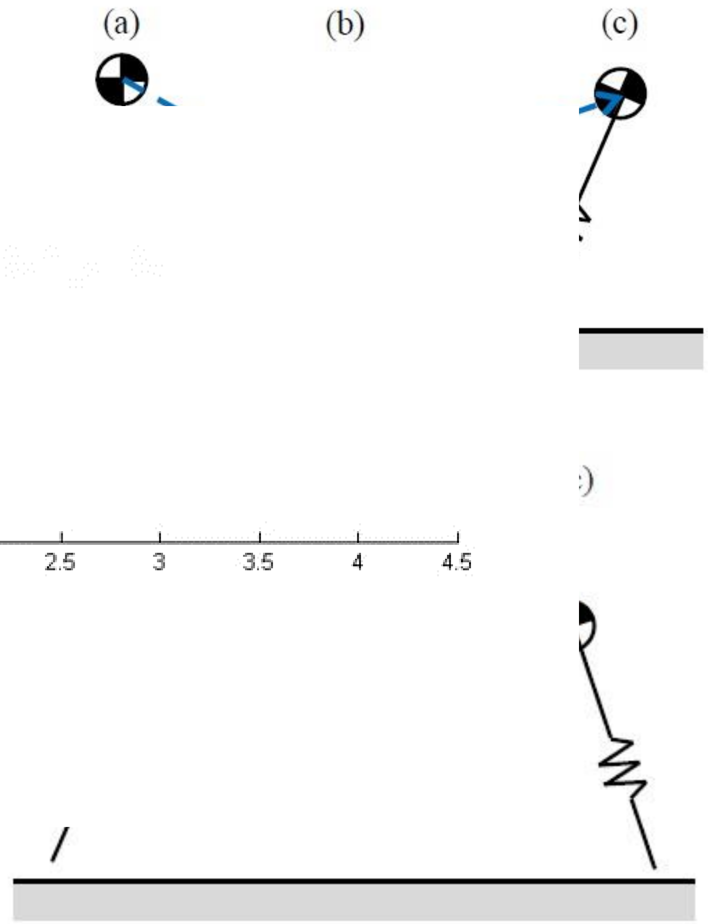
(a)->(b):Sprin

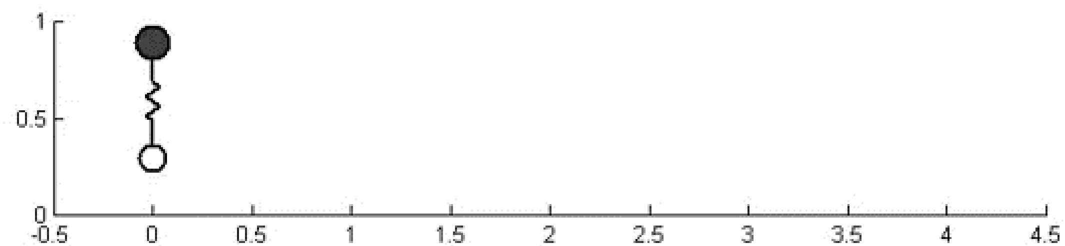
(b)->(c):Jumpi

- Flight phas

(a)->(b):Forwa

(b)->(c):Pullin

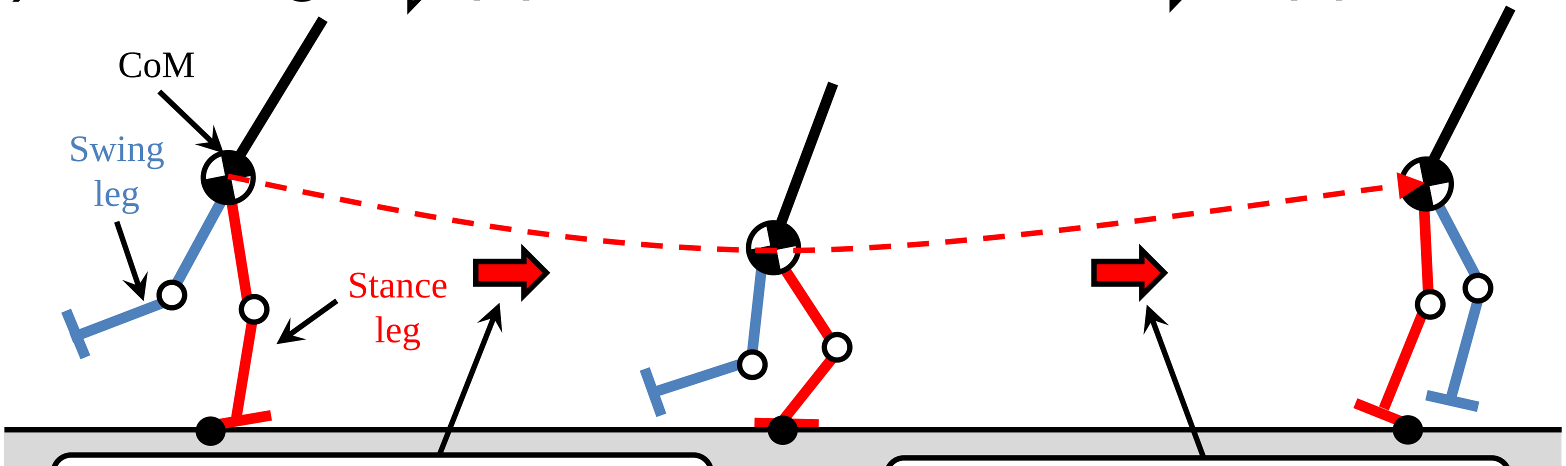




Control in Stance Phase (1)

Proposed running control

(a) Landing → (b) Bottom of CoM → (c) Takeoff



Suppress springs

Release springs

Accumulation of energy using robot dynamics

$$u_1 = 0, \quad u_2 = 0$$

More jumping force

$$u_1 = -K_{s1}(\theta_2 - \theta_1)$$

$$u_2 = -K_{s2}(\phi_{knee} + \theta_3 - \theta_2)$$

Control in Stance Phase (2)

Maintenance of body angle

$$u_3 = -K_{s3}(\theta_7 - \phi_{s3}) - D_{s3}\dot{\theta}_7 + u_4$$

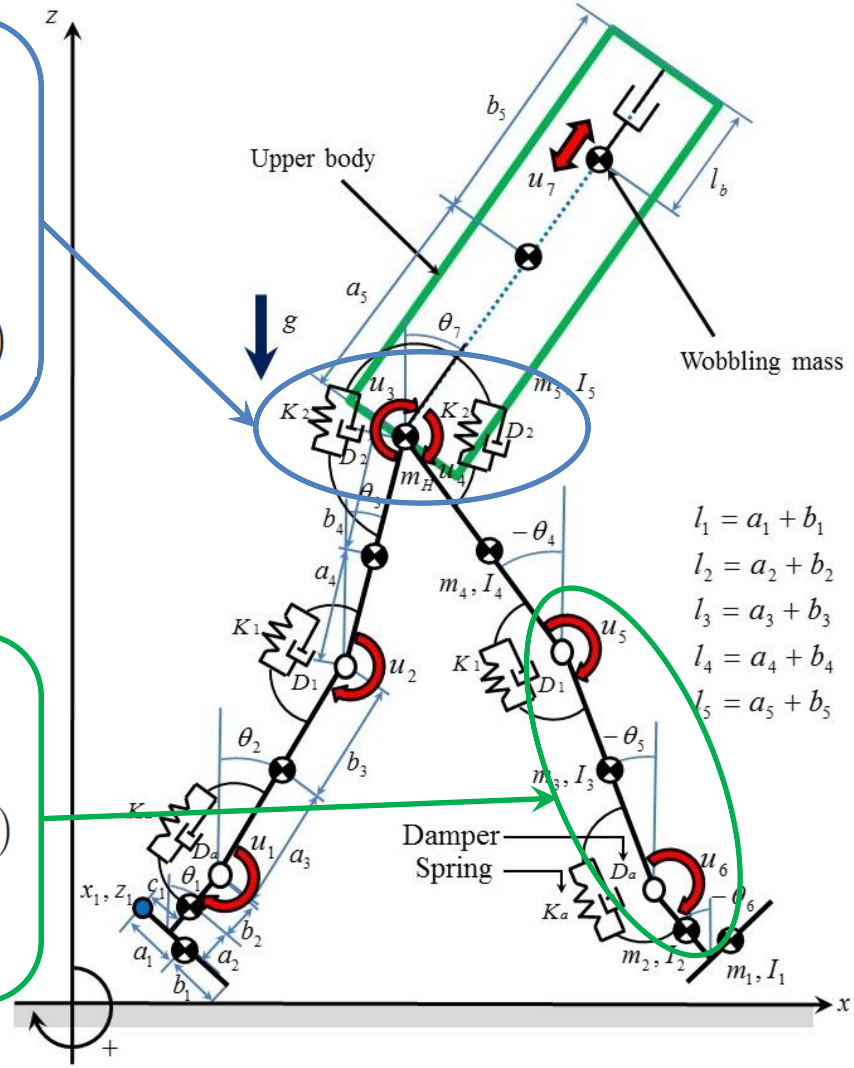
Swing up of swinging leg

$$u_4 = -K_{s4}(\phi_{s4} + \theta_4 - \theta_3) - D_{s4}(\dot{\theta}_4 - \dot{\theta}_3)$$

Avoidance of foot scuffing

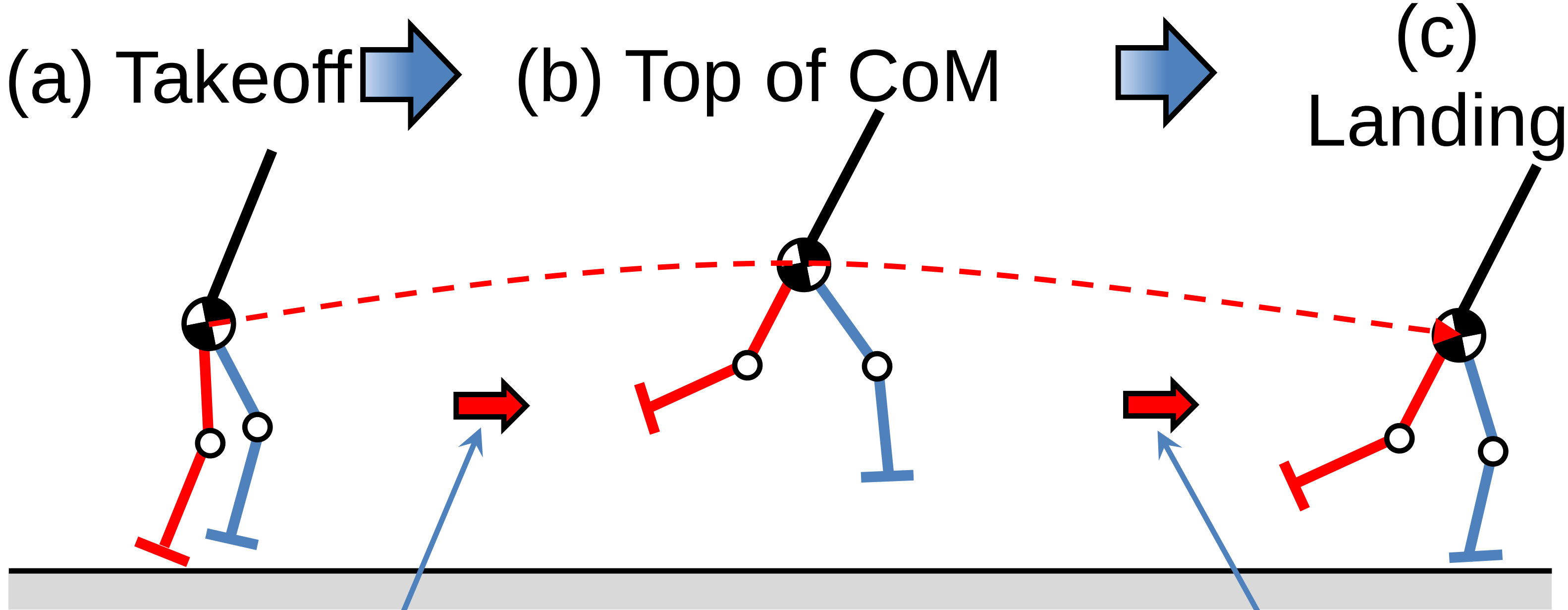
$$u_5 = -\tau_5 - K_{s5}(-\phi_{s5} + \theta_5 - \theta_4) - D_{s5}(\dot{\theta}_5 - \dot{\theta}_4)$$

$$u_6 = -\tau_6 - K_{s6}(\phi_{s6} + \theta_6 - \theta_5) - D_{s6}(\dot{\theta}_6 - \dot{\theta}_5)$$



Control in Flight Phase (1)

Proposed running control



Swing up of swinging leg

$$u_5 = -\tau_5 - K_{s5}(-\phi_{f5} + \theta_5 - \theta_4) - D_{s5}(\dot{\theta}_5 - \dot{\theta}_4)$$

$$u_6 = -\tau_6 - K_{s6}(\phi_{f6} + \theta_6 - \theta_5) - D_{s6}(\dot{\theta}_6 - \dot{\theta}_5)$$

Increase running stability

g

$$\phi_{f6} = \theta_5$$

Control in Flight Phase (2)

Maintenance of body angle

$$u_3 = -K_{s3}(\theta_7 - \phi_{s3}) - D_{s3}\dot{\theta}_7 + u_4$$

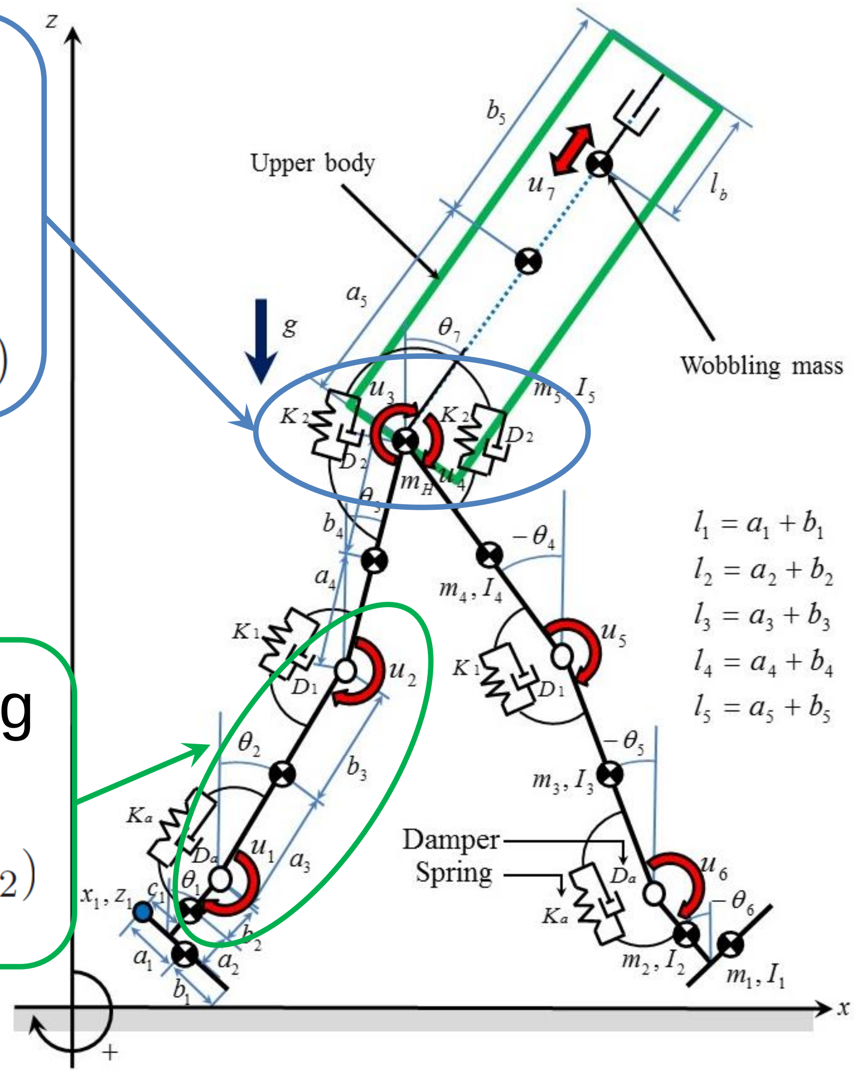
Swing up of swinging leg

$$u_4 = -K_{s4}(\phi_{s4} + \theta_4 - \theta_3) - D_{s4}(\dot{\theta}_4 - \dot{\theta}_3)$$

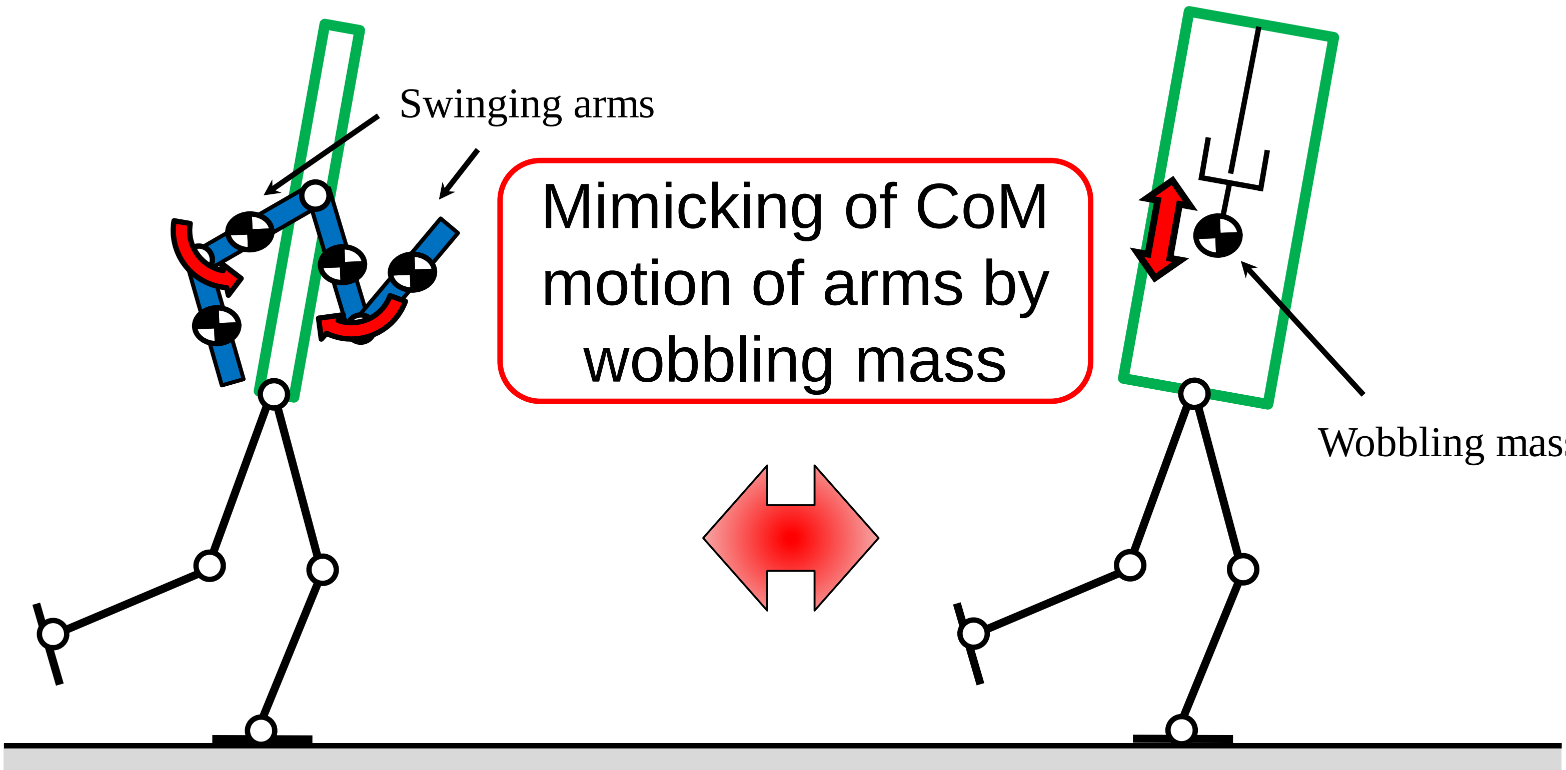
Avoidance of foot scuffing of rear leg

$$u_1 = 0$$

$$u_2 = -\tau_2 - K_{f2}(\phi_{f2} + \theta_3 - \theta_2) - D_{f2}(\dot{\theta}_3 - \dot{\theta}_2)$$



Control of Wobbling Mass (1)



Swinging arm in stance phase

Trajectory of CoM of arms is convex

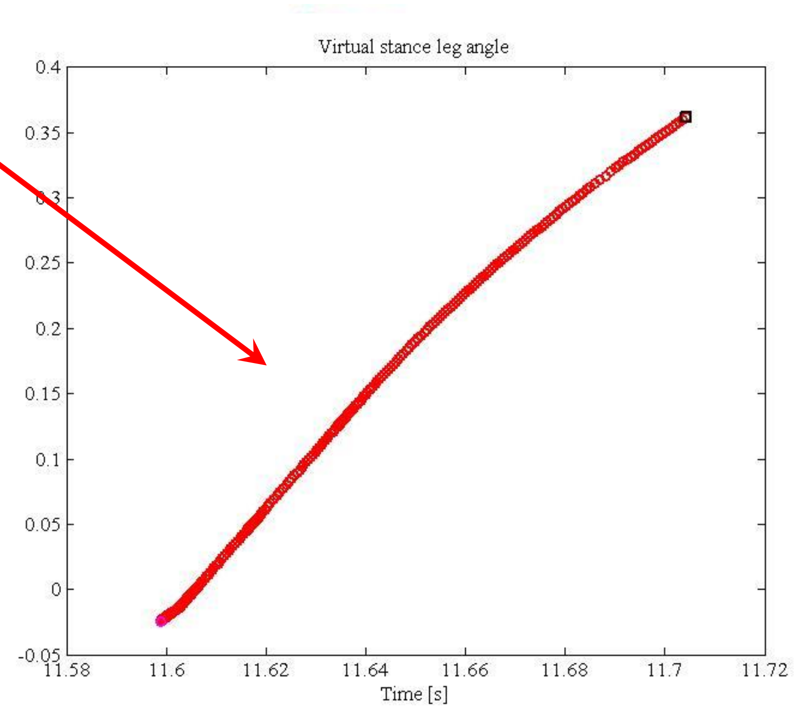
Design of trajectory of wobbling mass

Control of Wobbling Mass (2)

Angle of virtual stance leg

Monotonically increasing in stance phase

Timing parameter



Desired trajectory

Control law of wobbling mass

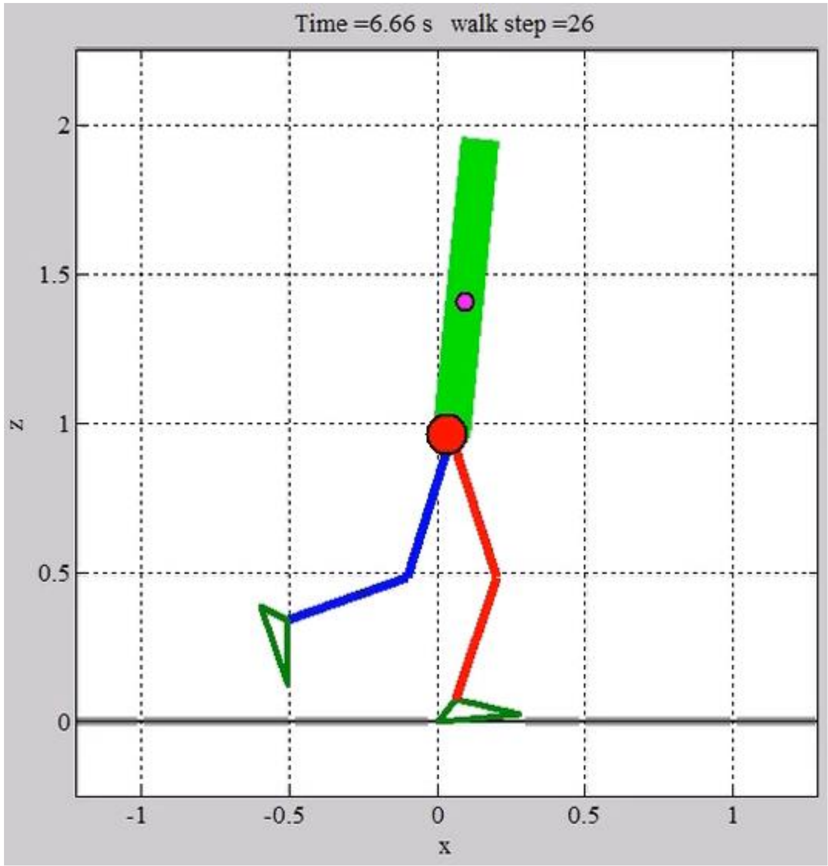
$$u_7 = -K_7(l_b - l_{bd}) - D_7(\dot{l}_b - \dot{l}_{bd})$$

$$l_{bd} = k_w \cos(\theta_{st} - \theta_o) + p_o$$

Simulation Result (1)

Faster running speed

	Locked	Wobbling (proposed method)
Converged speed [m/s]	3.95	4.94
Max speed [m/s]	4.08	5.22
Step time [s]	0.28	0.23

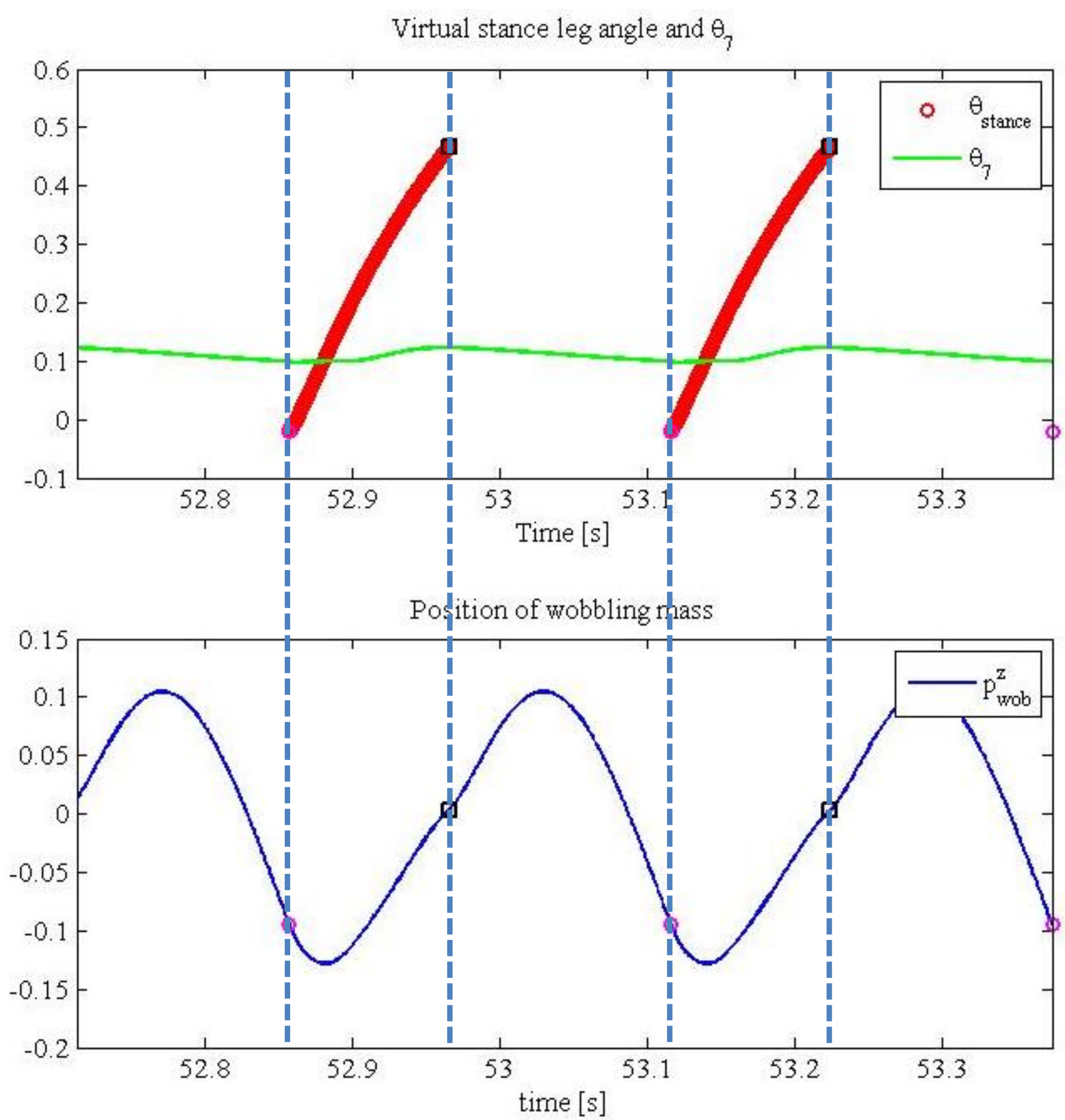


Simulation Result (2)

Monotonical increase of virtual stance leg angle

Convex trajectory of wobbling mass

Desired trajectory is realized



Mechanism of Faster Running (1)

$$\theta_{st} = \theta_7$$

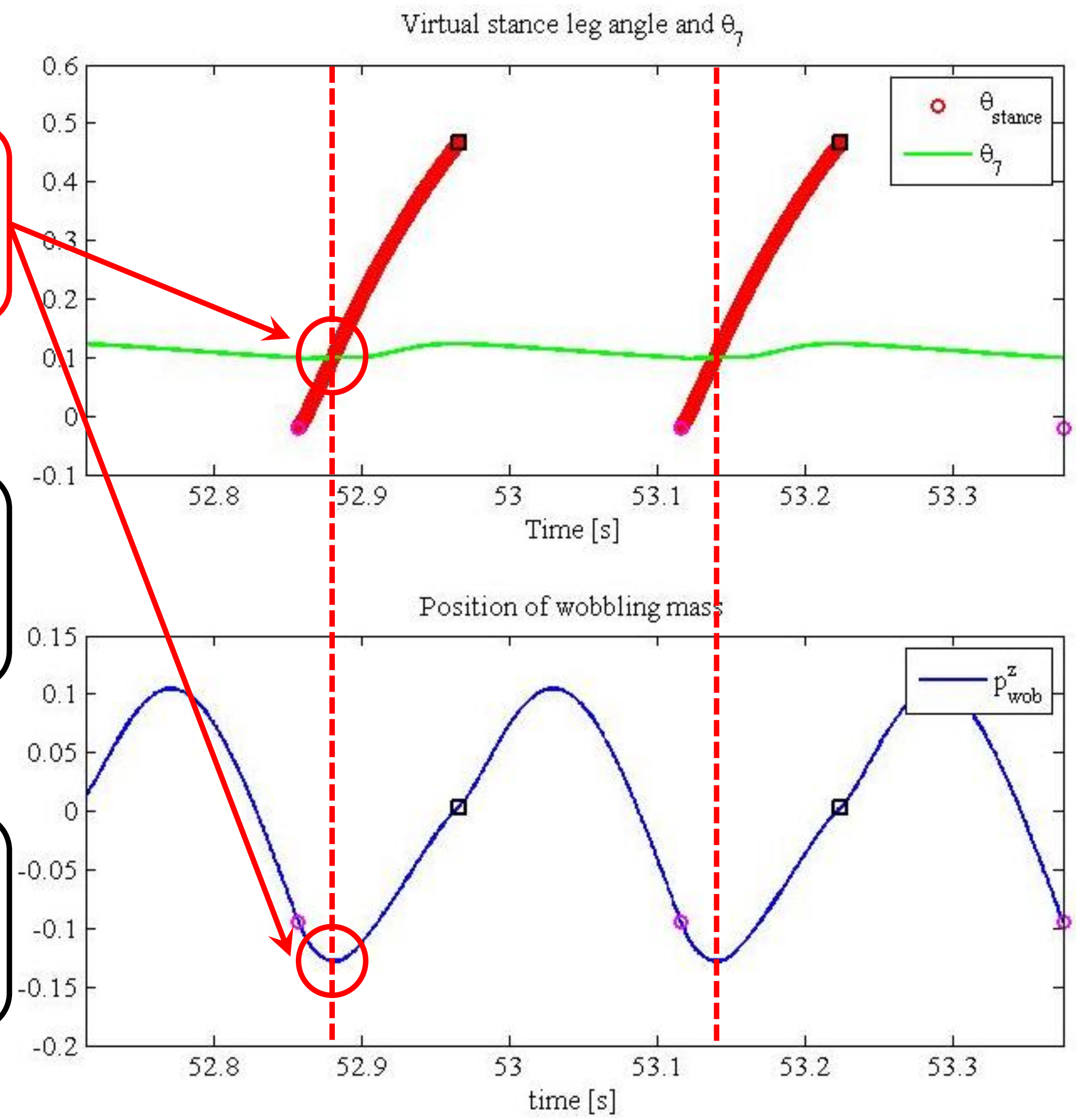
Bottom of wobbling mass

$$\theta_{st} < \theta_7$$

Down motion of the mass

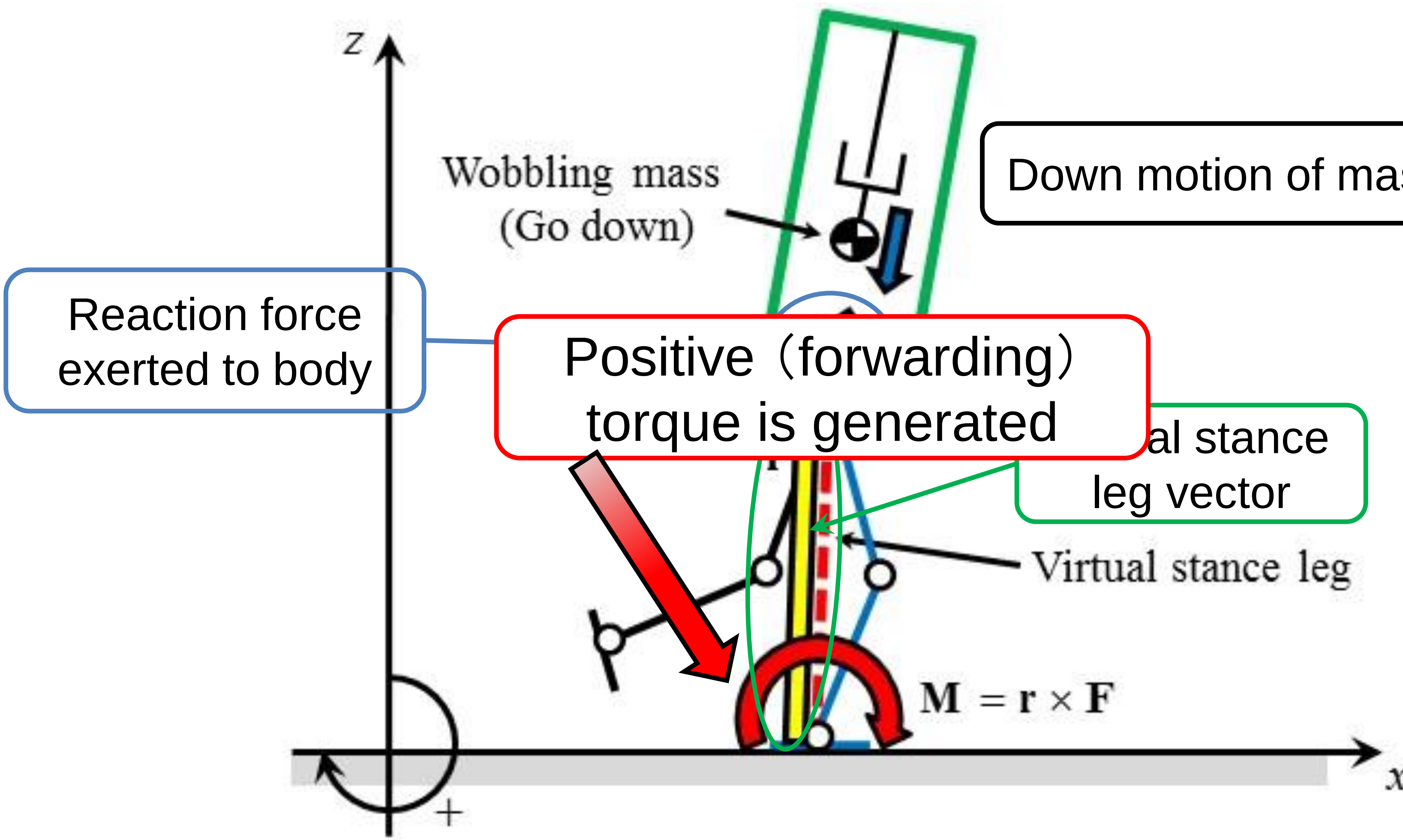
$$\theta_{st} > \theta_7$$

Up motion of the mass



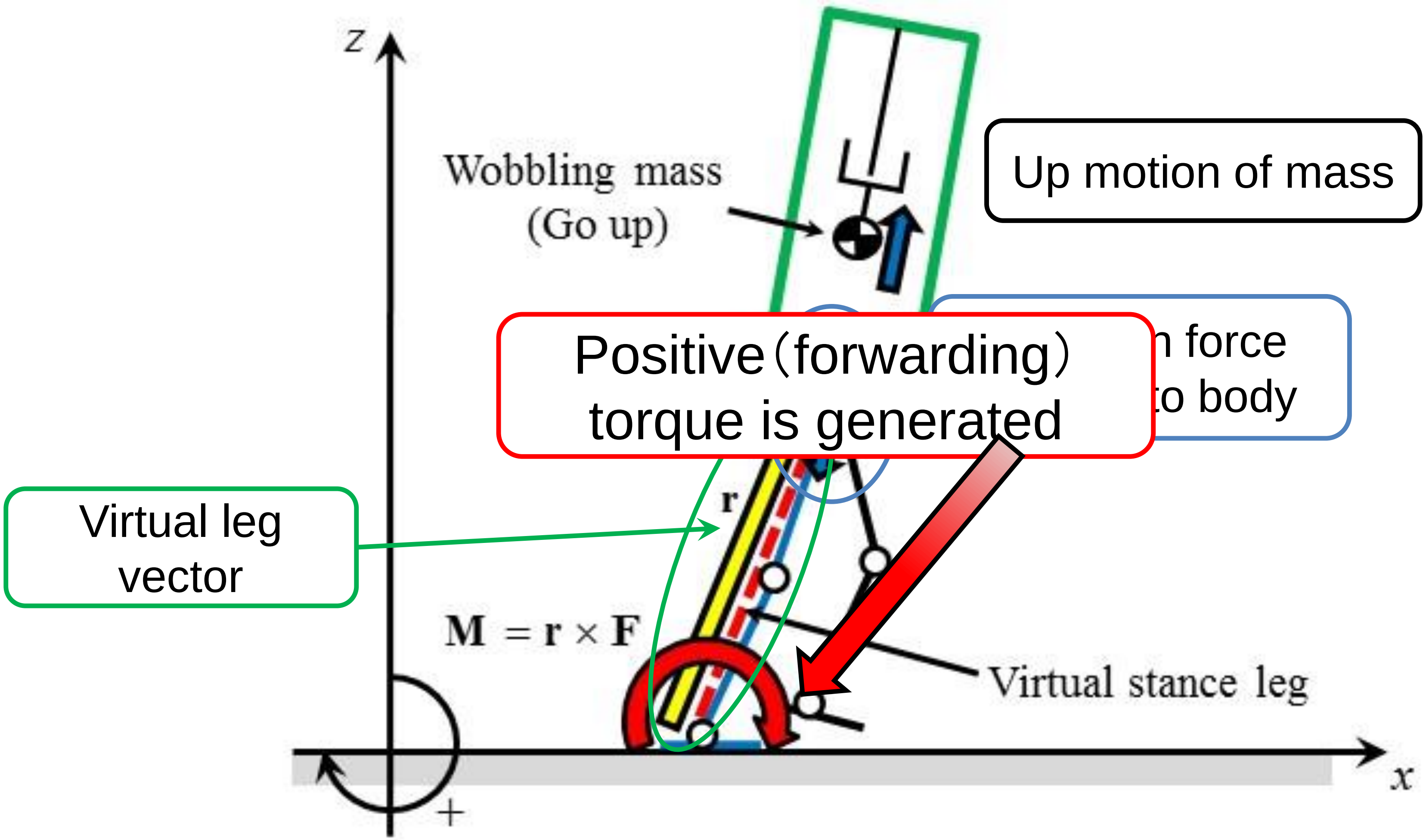
Mechanism of Faster Running (2)

$$\theta_{st} < \theta_7$$



Mechanism of Faster Running (3)

$$\boxed{\theta_{st} \quad > \quad \theta_7}$$

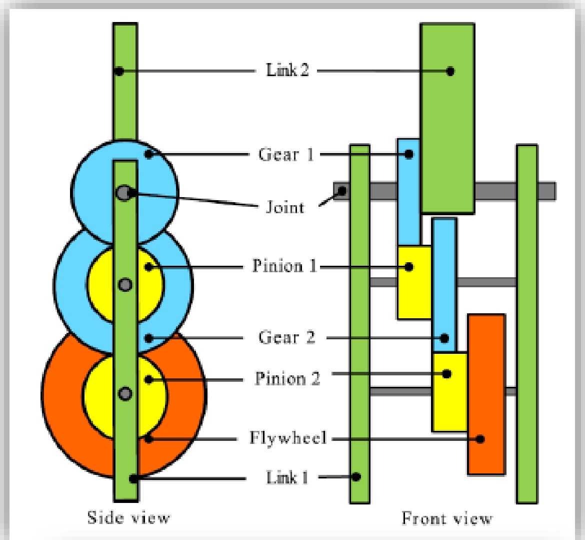


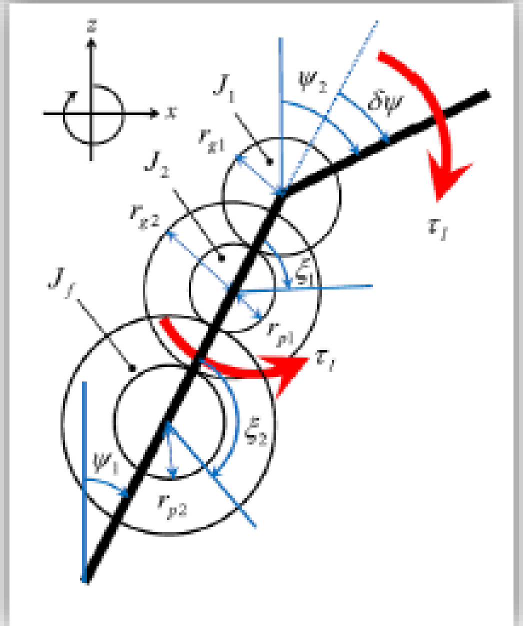
Effect of Inerter for Running without Wolbbing Mass

- Addition of inertia without heavy mass in addition to spring and damping, whose reaction torque is proportional to angular acceleration
  - 2 links, 2 gears, 2 pinions, and 1 flywheel
  - 2 links are rotated around a joint
  - Link 2 and gear 1, pinion 1 and gear 2, pinion 2 and flywheel are attached on a common shaft, respectively
- Dynamics of inerter

$$\tau_I = \beta(\sigma\ddot{\psi}) = \beta(\ddot{\psi}_2 - \ddot{\psi}_1)$$

$$\beta = J_f\alpha_1^2\alpha_2^2[\mathrm{Nm}/(\mathrm{rad}/\mathrm{s}^2)]$$





Performance Indexes

- Specific resistance: index to evaluate energy efficiency of movement

$$SR := \frac{p}{mgv}$$

$$p = \frac{1}{T}\int_0^T \left(|u_1(\dot\theta_2-\dot\theta_1)| + |u_2(\dot\theta_3-\dot\theta_1)| + |u_3(\dot\theta_7-\dot\theta_3)| + |u_4(\dot\theta_4-\dot\theta_7)| + |u_5(\dot\theta_5-\dot\theta_5)| + |u_6(\dot\theta_6-\dot\theta_5)|\right) dt$$

where p はaverage energy, m is a total mass, v is a moving velocity

Smaller is S R , better is energy efficiency

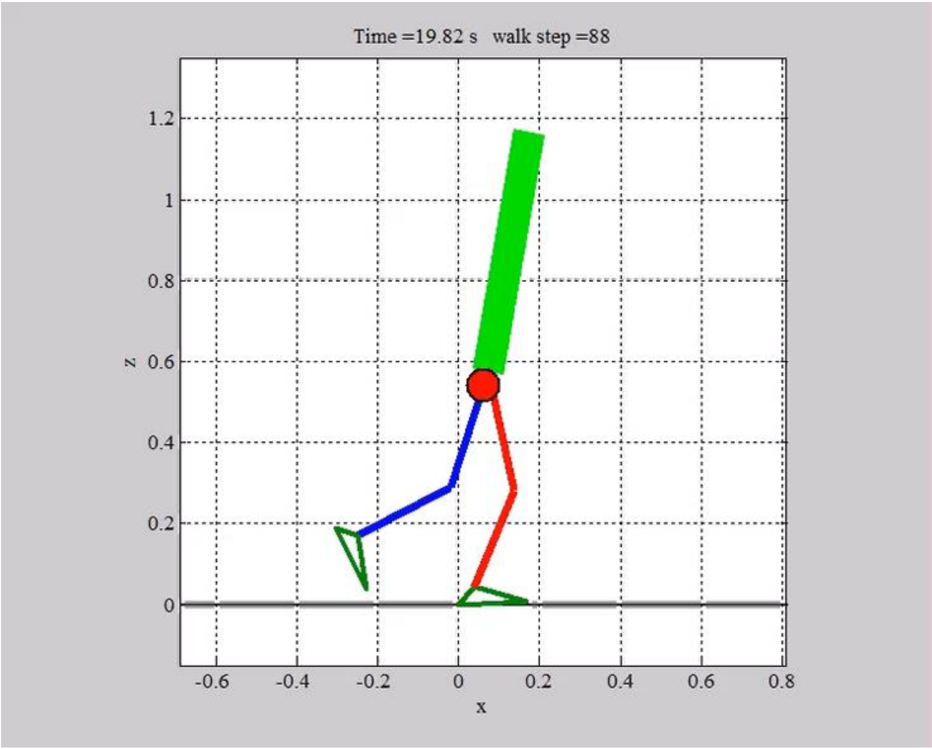
- Froude number: normalized index to evaluate moving velocity independent of size

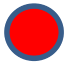
$$F := \frac{v}{\sqrt{gl}}$$

where g is the gravity constant, l is a leg length

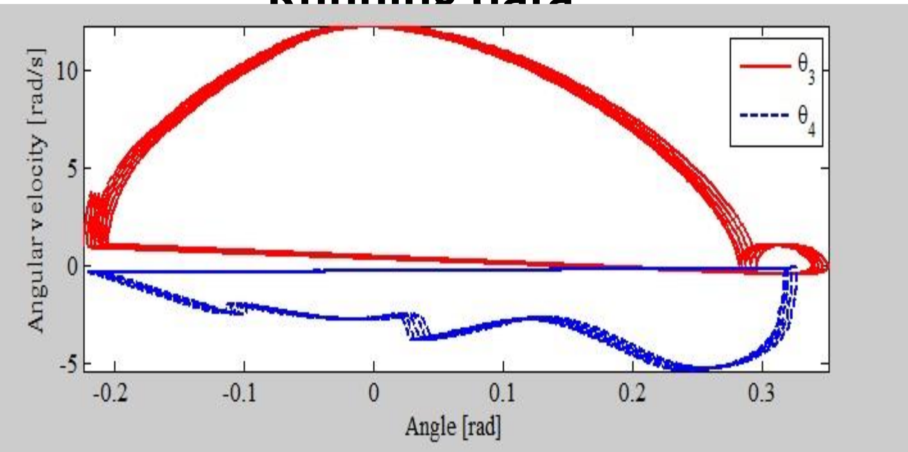
Bigger is F , faster is moving velocity

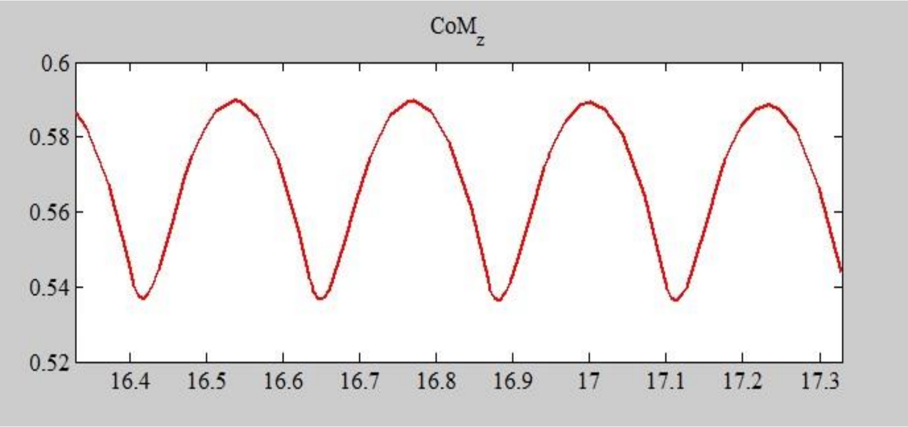
Simulation Result without Wobbling mass (1)





Running data





A biped robot based on SLIP model realized more energy efficient and faster running than ASIMO based on ZMP

Simulation Result without Wobbling mass (2)

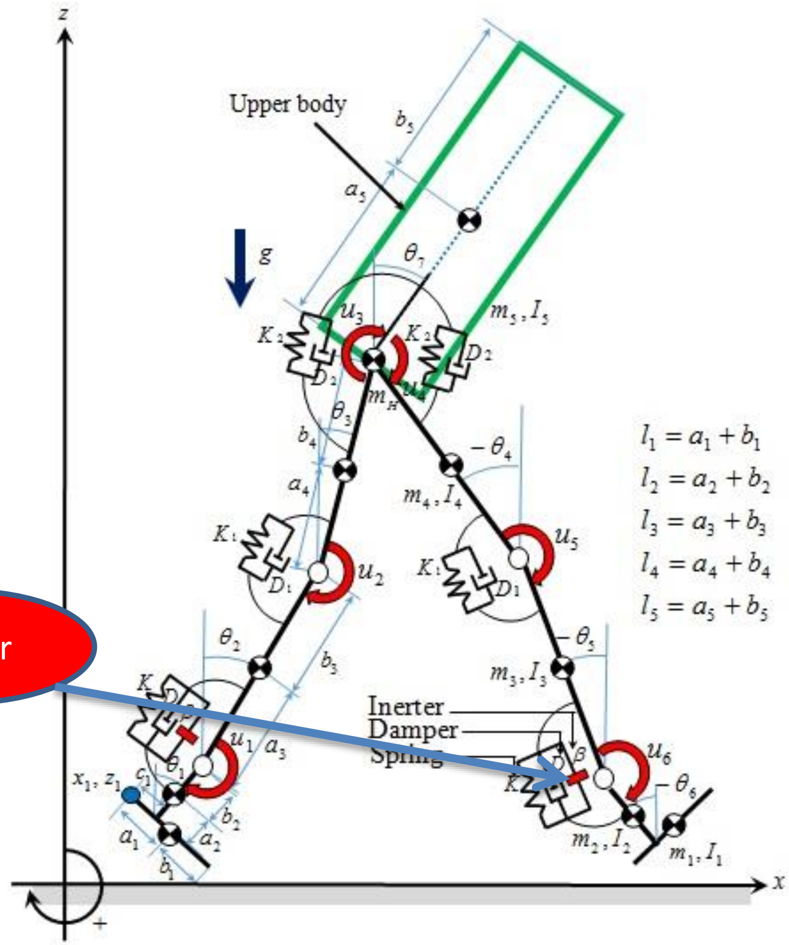
Generalized coordinate vector

$$\boldsymbol{q} = [\theta_1,\ \theta_2,\ \theta_3, \theta_4,\ \theta_5,\ \theta_6,\ \theta_7,\ x_1,\ z_1]^{\mathrm{T}}$$

Dynamic equation

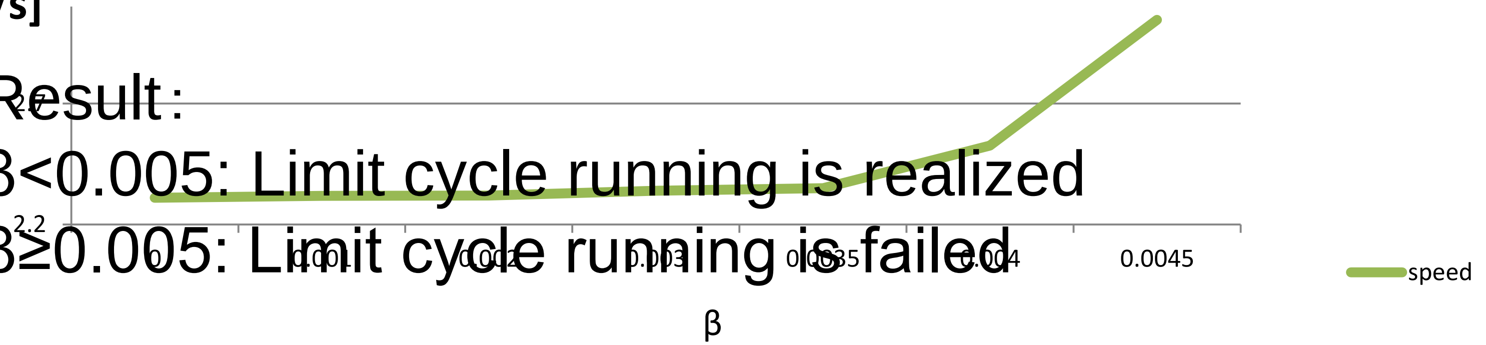
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = S_1 u + S_2 \tau - \beta_m \ddot{q} + J_c(q)^{\mathrm{T}} \lambda$$

inerter



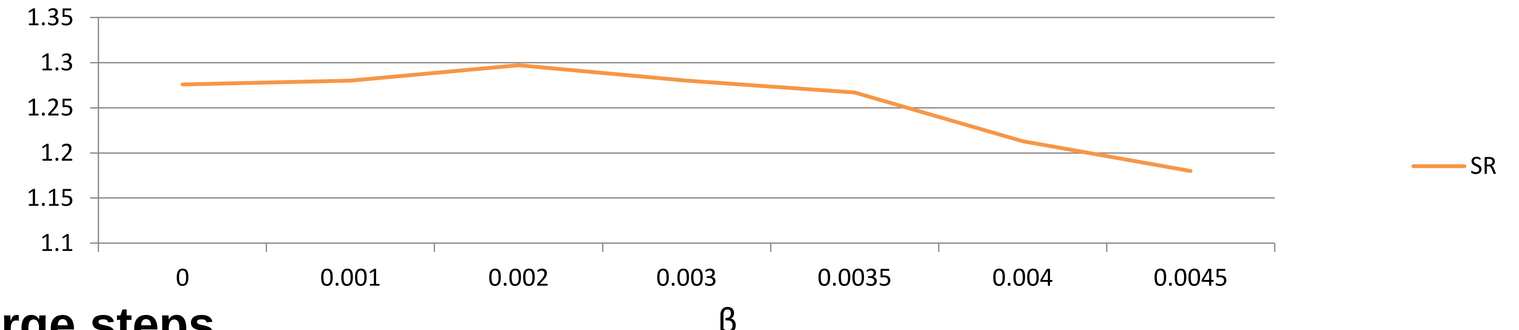
Simulation Resultwithout Wobbling mass (3)

Speed[m/s]

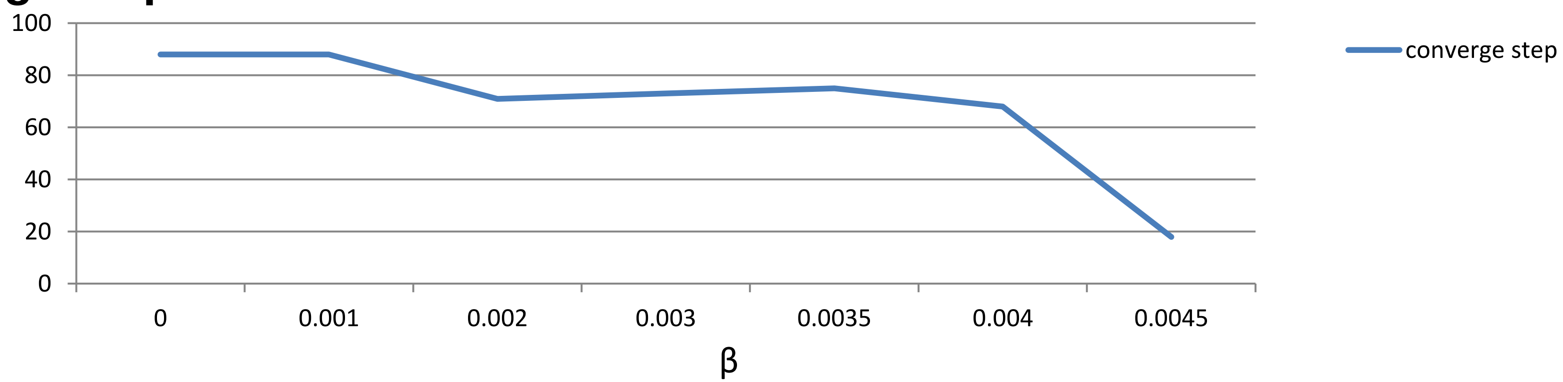


Result：
β<0.005: Limit cycle running is realized
β≥0.005: Limit cycle running is failed

SR



Converge steps



Simulation Result without Wobbling mass (4)

β	Speed[m/s]	Converge Steps	SR
0.000	2.3108	88	1.2768
0.001	2.3184	88	1.2809
0.002	2.3192	71	1.2974
0.003	2.3395	73	1.2869
0.0035	2.3507	75	1.2673
0.004	2.5257	68	1.2134
0.0045	3.0464	18	1.1879
0.005	*fail*	*fail*	*fail*

Qualitative Properties

- β : [0.000,0.005]

β ↑ → Running speed ↑, Convergence rate ↑, Energy efficiency ↑

β	0.000	0.0045
V[m/s]	2.31	3.04
step	88	18
SR	1.276	1.18
F	0.95	1.25

Speed is increased by 30%
SR is decreased by 8%

Inerter can increase running performances of biped robots based on SLIP model

Effect of Inerter for Running with Wobbling Mass

Wobbling mass	β	Speed	Convergence	SR
off	0	2.31	88	1.28
off	0.0045	3.05	18	1.19

Obtained Result 3

- Wobbling mass and Inerter individually can improve performances of biped running robot based on SLIP model.

- Future work
  - Verification of the effect of both wobbling mass and ineters for running control of biped robots
  - Verification by experimental robots
  - Optimization of physical parameters

Acknowlegment

I give my special thanks to my ex-students

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Yu Iemura

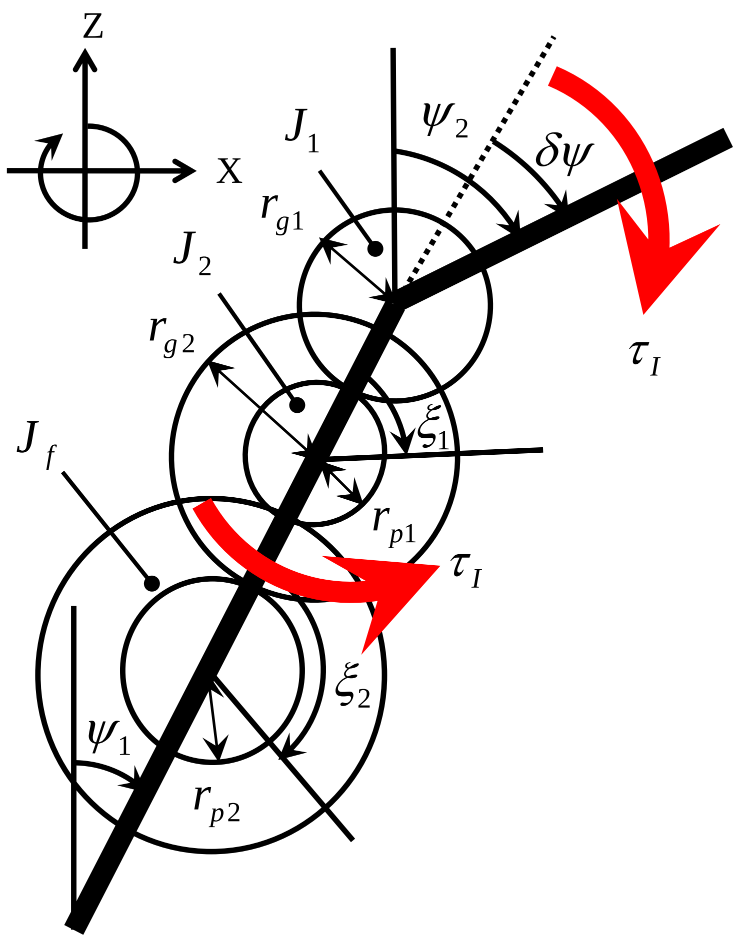
Doan Nat Thanh

Thank you for your attention!

Appendix

- Inerter detail and movie.
- Detail of walking experiment.
- All experiment result.
- Virtual ankle torque.

Derivation of Dynamic Equation of the Rotary Inerter



Model of the rotary inerter

We derive three dynamic equations

$$J_1\ddot{\psi}_2 = \tau_I - r_{g1}F_1$$

$$J_2\ddot{\xi}_1 = r_{p1}F_1 - r_{g2}F_2$$

$$(J_3 + J_f)\ddot{\xi}_2 = r_{p2}F_2$$

J_i : *Inertia of gear etc.*

r_{pi} : *Radius of pinion.*

r_{gi} : *Radius of gear.*

ψ_i : *Link angle.*

ξ_i : *Rotation angle of gear.*

F_i : *Force between gear and pinion*

We arrange above three equations

$$\tau_I = J_f\left(\frac{r_{g1}r_{g2}}{r_{p1}r_{p2}}\right)^2(\ddot{\psi}_2 - \ddot{\psi}_1)$$

We define that rg_1/rp_1 is α_1 and rg_2/rp_2 is α_2

$$\tau_I = \boxed{J_f\alpha_1^2\alpha_2^2}(\ddot{\psi}_2 - \ddot{\psi}_1)$$

Inerter Constant : $\beta = \boxed{J_f}\,\boxed{\alpha_1^2\ \alpha_2^2}$

Inertia of the flywheel Gear ratio of Inerter

Movie of Ankle Inerter

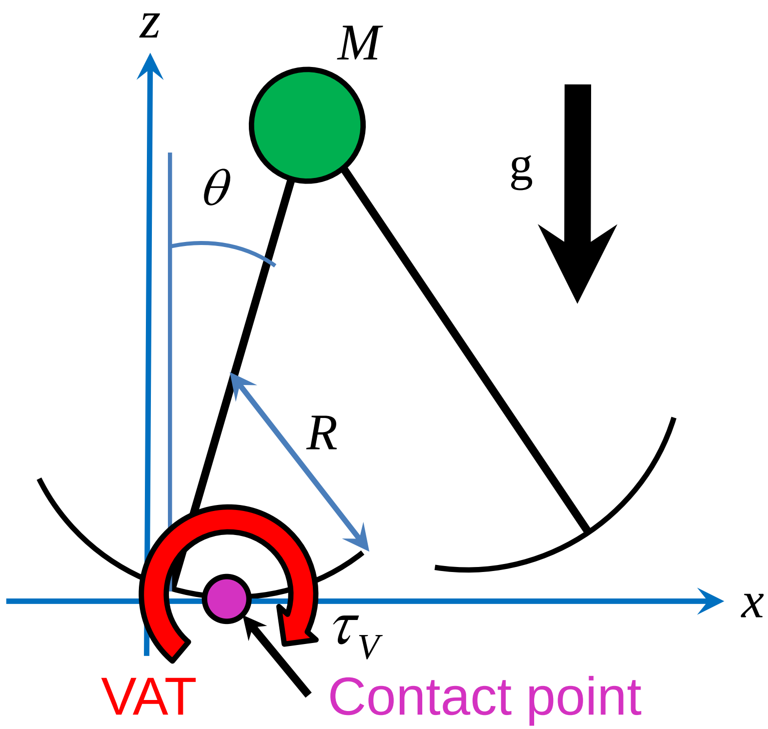
Without Inerter

With Inerter

We can see that effect of the Inerter.

Virtual Ankle Torque (VAT)

Arc-footed biped robots achieve high-speed and high-efficient walking by virtual ankle torque.(F.Asano 2006)



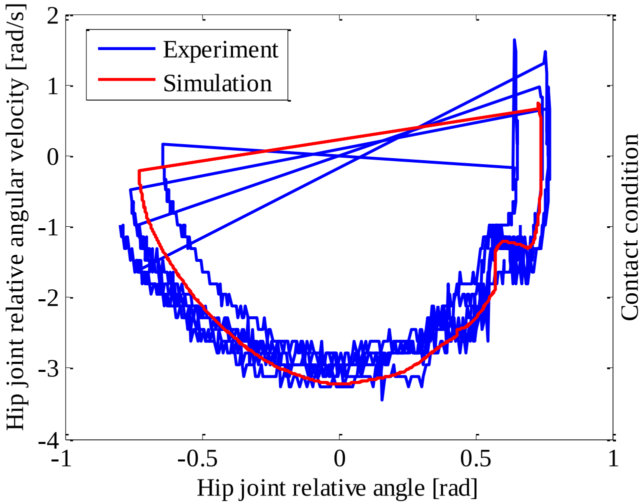
VAT is virtual torque around contact point of arc-footed biped robot

$$\tau_V = -MRg\sin\theta \approx -MRg\theta$$

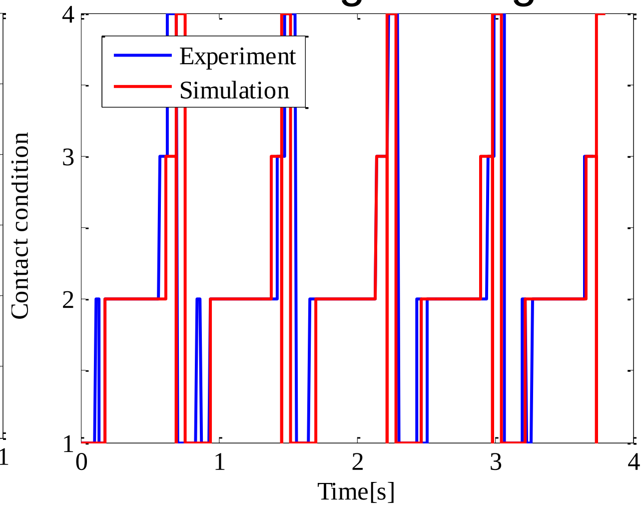
VAT has linear relationship to support leg angle

Experiment & Simulation result

Phase plane of the hip joint



Transition of contact condition during walking



1 : Heel-Contact 2 : Foot-Flat
3: Toe-Contact 4: Double-Support

Walking speed & period

	SIM	Experiment
Speed[m/s]	0.54	0.53
Period[s]	0.76	0.77

Phase trajectory converges the limit cycle of the simulation

Transition of contact condition is equal to simulation result.

Walking speed & period are equal to simulation result

Our biped walker achieves walking which is equal to the simulation result

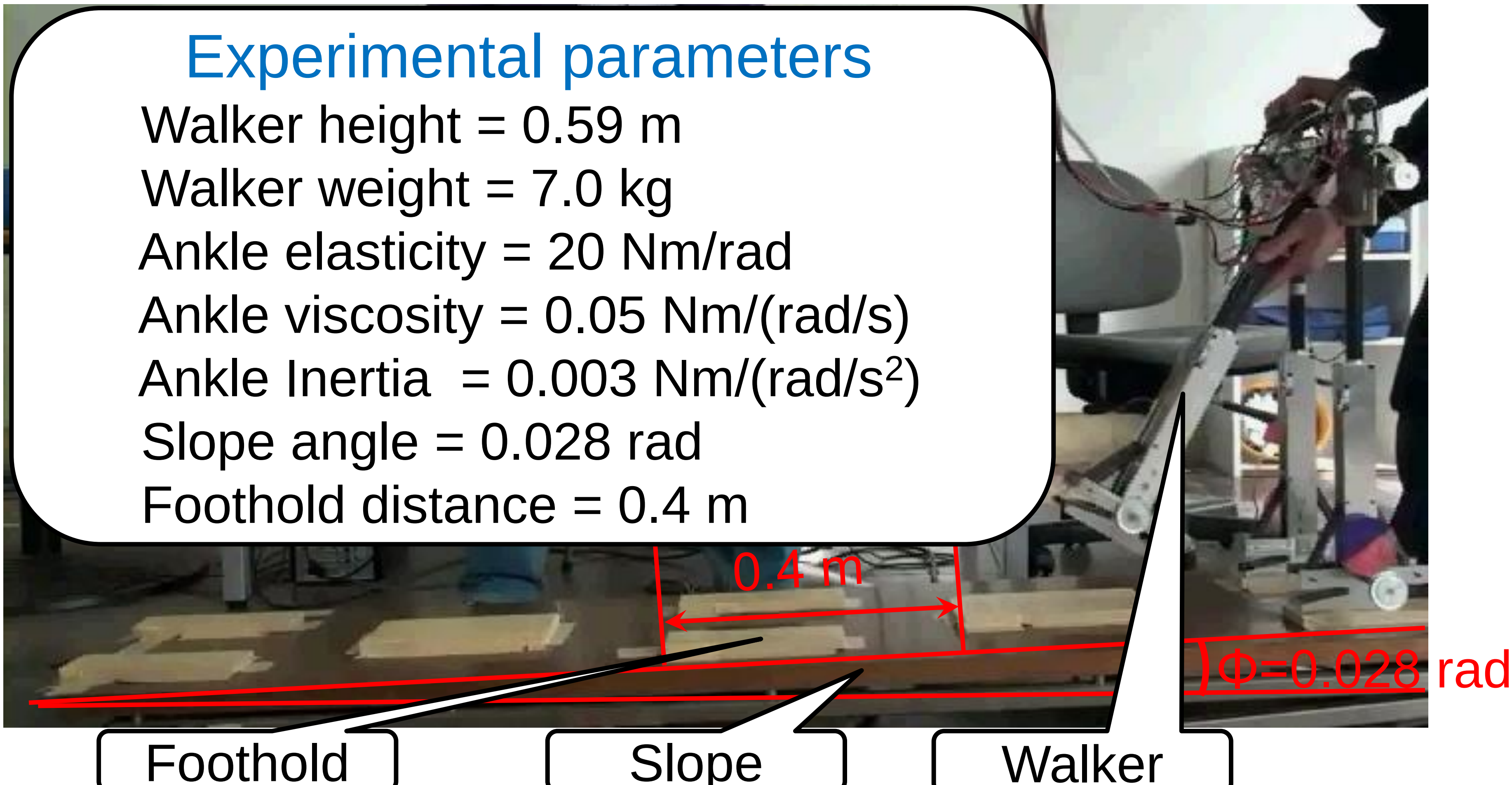
Our model of biped walker and previous simulation results are valid.

Experimental Condition of Passive Walking

To confirm validity of walking simulation,
we carry out walking experiment.

Experimental parameters

Walker height = 0.59 m
Walker weight = 7.0 kg
Ankle elasticity = 20 Nm/rad
Ankle viscosity = 0.05 Nm/(rad/s)
Ankle Inertia = 0.003 Nm/(rad/s^2)
Slope angle = 0.028 rad
Foothold distance = 0.4 m



Experimental environment

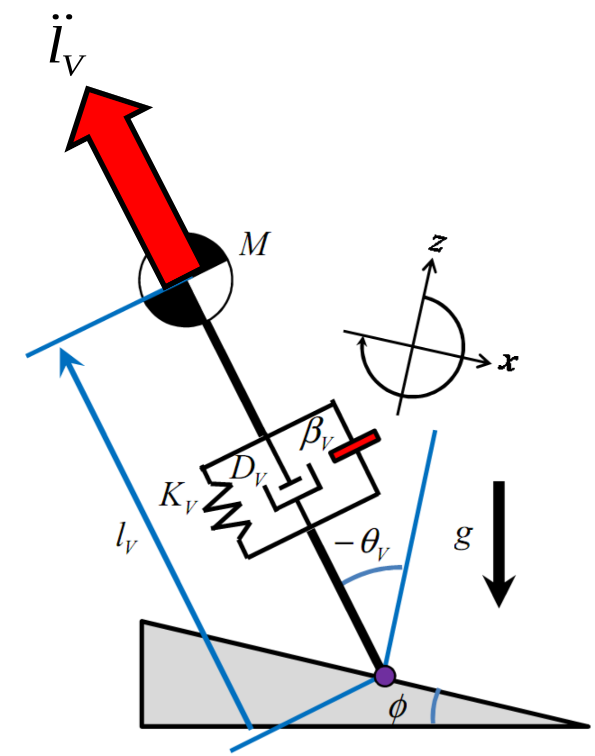
Effect of Ankle Viscosity and Inertia

Centrifugal force　Elastic force　Viscous force

$$\ddot{l}_V = \frac{\boxed{M l_V \theta_V^2} \boxed{- K_V(l_V - l_{V0})} \boxed{- D_V \dot{l}_V} - Mg\cos(\theta_V + \phi)}{Mg + \boxed{\beta_V}}$$

Inertia by Ankle Inerter

Model of Heel Contact Phase



$\ddot{l}_V > 0$ cause floating (slipping) biped robot

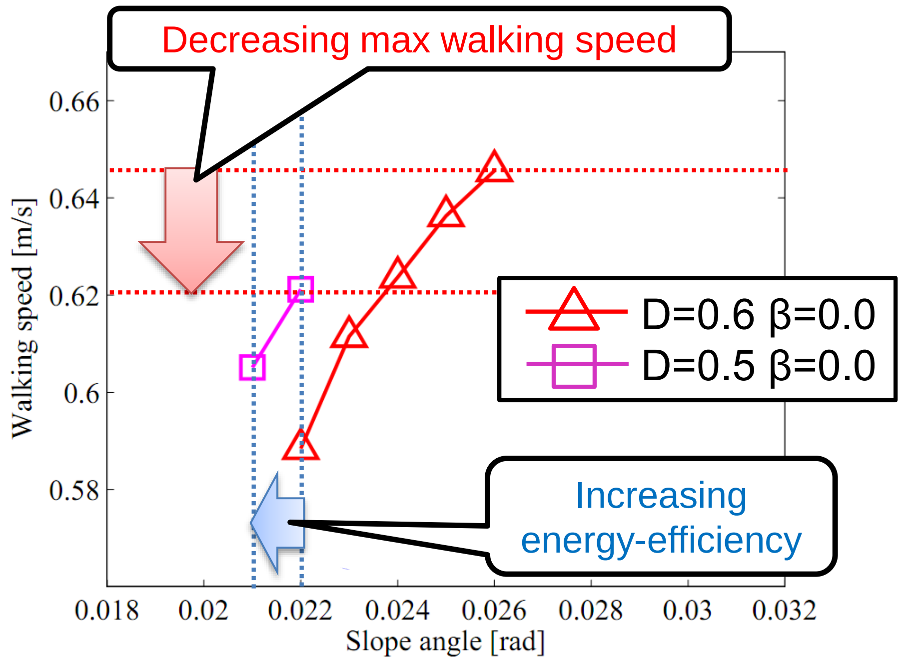
To achieve high-speed walking on steep slope, increasing $\ddot{l}_V$ is restricted.

By ankle viscosity and inertia,
We can achieve high-speed walking of the robots.

Energy-loss of ankle viscosity is not equal to that of ankle inertia.
Difference is Effect of restrictioning acceleration with respect to Energy-loss in viscosity and inertia.

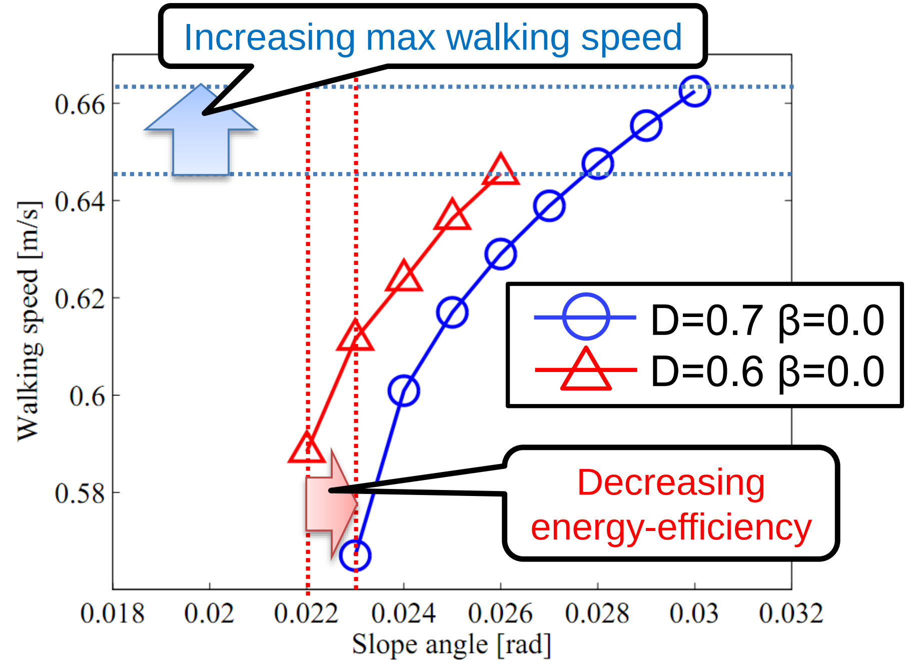
Trade-off of Walking Performance

To improve energy efficiency of walking of the biped robot, we decrease ankle viscosity.



- Energy-efficiency increases.
- Max walking speed decreases.

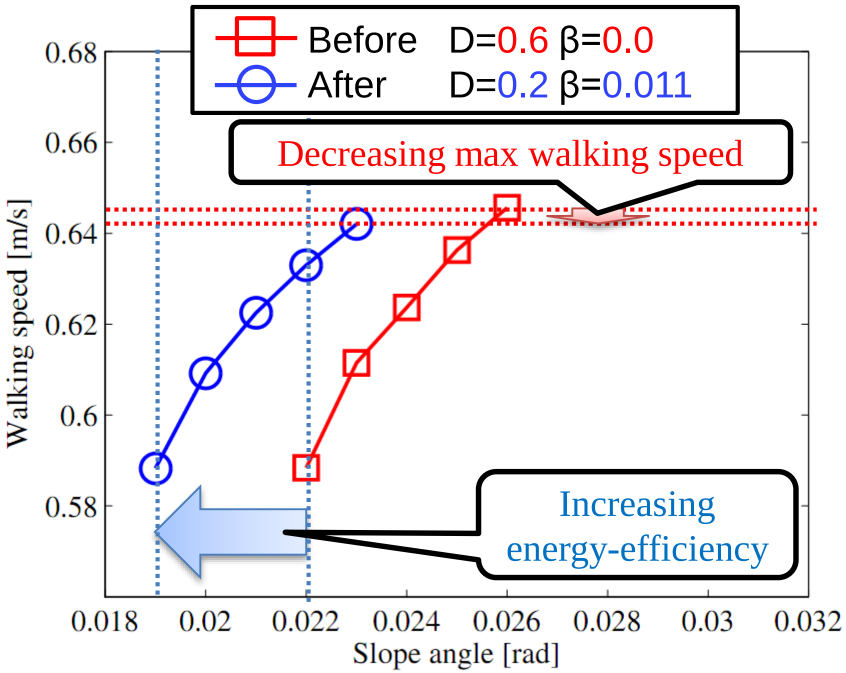
To improve max walking speed of the biped robot, we increase ankle viscosity.



- Max-walking speed increases.
- Energy-efficiency decreases.

we cannot improve both energy efficiency and walking speed.

Achieving efficient walking by the ankle Inerter



By optimization of ankle viscosity and inertia for energy efficiency, we consider improving energy efficiency of walking

In the biped robot with ankle inerter, we can easily design ankle inertia.

We change ankle viscosity and inertia.

D = 0.6 Nm/(rad/s) β = 0.0 Nm/(rad/s^2)

D = 0.2 Nm/(rad/s) β = 0.011Nm/(rad/s^2)

The biped robot with ankle inerter achieve more energy-efficient walking, almost without decreasing max walking speed.